PLAN AND PREPARE

Main Ideas

In this chapter the students will classify triangles, find measures of angles of triangles, identify congruent figures, and prove triangles congruent. They will also use theorems about isosceles and equilateral triangles and perform transformations.

Prerequisite Skills

Skills Readiness, available online, provides review and practice for the Skills and Algebra Check portion of the Prerequisite Skills quiz.

How student answers the exercises	What to assign from *Skills Readiness*
Any of Exs. 5–7 answered incorrectly	**Skill 69** Solve multi-step equations
Any of Exs. 8–10 answered incorrectly	**Skill 73** Find the midpoint
Any of Exs. 11–14 answered incorrectly	**Skill 26** Use theorems about parallel lines
All exercises answered correctly	Chapter Enrichment

Additional skills review and practice is available in the Skills Review Handbook and the @HomeTutor.

4 Congruent Triangles

Before

Previously, you learned the following skills, which you'll use in this chapter: classifying angles, solving linear equations, finding midpoints, and using angle relationships.

Prerequisite Skills

VOCABULARY CHECK

Classify the angle as *acute*, *obtuse*, *right*, or *straight*.

1. $m\angle A = 115°$ obtuse **2.** $m\angle B = 90°$ right **3.** $m\angle C = 35°$ acute **4.** $m\angle D = 95°$ obtuse

SKILLS AND ALGEBRA CHECK

Solve the equation.

5. $70 + 2y = 180$ 55 **6.** $2x = 5x - 54$ 18 **7.** $40 + x + 65 = 180$ 75

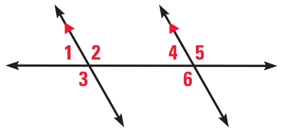
Find the coordinates of the midpoint of $\overline{PQ}$. 8–10. See margin.

8. $P(2, -5), Q(-1, -2)$ **9.** $P(-4, 7), Q(1, -5)$ **10.** $P(h, k), Q(h, 0)$

Determine whether the angles are congruent. If so, explain why. 11–14. See margin.

11. $\angle 2, \angle 3$ **12.** $\angle 1, \angle 4$

13. $\angle 2, \angle 6$ **14.** $\angle 3, \angle 4$



Chapter Planning Guide

Chapter Resource Book
- Teaching Guide/Lesson Plan
- Project with Rubric

Assessment and Intervention
- Assessment Book
- Benchmark Tests
- Remediation Book
- Skills Readiness

Interactive Technology
- Power Presentations
- Activity Generator
- Animated Geometry
- ExamView™ Assessment Suite
- Online Quizzes
- eEdition
- @HomeTutor

Resources for English Learners
- Spanish Study Guide
- Multi-Language Visual Glossary
- Student Resources in Spanish

Now

In this chapter, you will apply the big ideas listed below and reviewed in the Chapter Summary. You will also use the key vocabulary listed below.

Big Ideas

1. **Classifying triangles by sides and angles**
2. **Proving that triangles are congruent**
3. **Using coordinate geometry to investigate triangle relationships**

Key Vocabulary

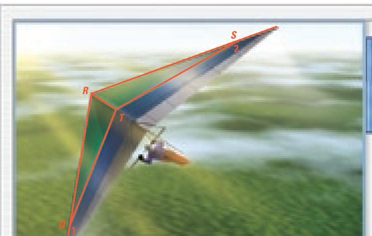
- triangle
  scalene, isosceles, equilateral, acute, right, obtuse, equiangular
- interior angles
- exterior angles
- corollary
- congruent figures
- corresponding parts
- rigid motion
- right triangle
  legs, hypotenuse
- flow proof
- isosceles triangle
  legs, vertex angle, base, base angles
- transformation translation, reflection, rotation

Why?

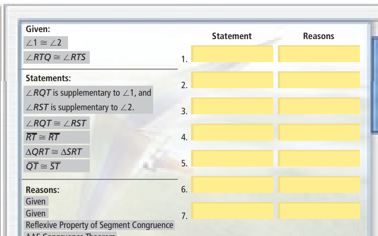
Triangles are used to add strength to structures in real-world situations. For example, the frame of a hang glider involves several triangles.

Animated Geometry

The animation illustrated below helps you answer a question from this chapter: What must be true about $\overline{QT}$ and $\overline{ST}$ for the hang glider to fly straight?



You will use congruent segments and angles in the hang glider to write a proof.



Scroll down to see the information needed to prove that $\overline{QT} \cong \overline{ST}$.

Animated Geometry at my.hrw.com

Differentiated Instruction Resources

- Reading Strategies
- Differentiated Instruction Lesson Notes
- English Learners Lesson Notes
- Inclusion Lesson Notes
- Teaching Strategies with Sample Worksheets
- Using Technology in the Classroom
- Tips for New Teachers
- Math Background Notes
- Assessment Strategies
- Teacher Survival Activities
- Bulletin Board Idea

8. $\left(\frac{1}{2}, -\frac{7}{2}\right)$

9. $\left(-\frac{3}{2}, 1\right)$

10. $\left(h, \frac{k}{2}\right)$

11. Yes; Vertical Angles Congruence Theorem

12. Yes; Corresponding Angles Postulate

13. Yes; Alternate Interior Angles Theorem

14. No

205

1 PLAN AND PREPARE

Explore the Concept

- Students will find the sum of the measures of three angles of a triangle.
- This activity leads into the study of the Triangle Sum Theorem in this lesson.

Materials

Each student will need:

- scissors
- ruler

Recommended Time

Work activity: 10 min

Discuss results: 5 min

Grouping

Students should work individually.

2 TEACH

Tips for Success

Be sure students arrange the corners with the vertices at the same point. Have them place them on a line on lined paper so the relationship is easy to see.

Key Question

- Does it matter what kind of triangle you use? **no**

Alternative Strategy

Demonstrate on an overhead projector with triangles drawn on transparencies.

Key Discovery

The sum of the measures of the angles of a triangle is 180°.

3 ASSESS AND RETEACH

1. Given the measures of three angles, how can you tell whether they could be the measures of the angles of a triangle? **Check to see whether their sum is 180°.**

Investigating Geometry ACTIVITY *Use before Apply Triangle Sum Properties*

Angle Sums in Triangles

MATERIALS • paper • pencil • scissors • ruler

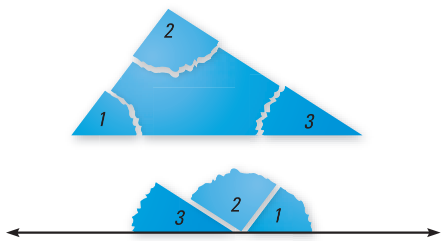
QUESTION **What are some relationships among the *interior angles* of a triangle and *exterior angles* of a triangle?**

EXPLORE 1 **Find the sum of the measures of interior angles**

STEP 1 ***Draw triangles*** Draw and cut out several different triangles.

STEP 2 ***Tear off corners*** For each triangle, tear off the three corners and place them next to each other, as shown in the diagram.

STEP 3 ***Make a conjecture*** Make a conjecture about the sum of the measures of the interior angles of a triangle.



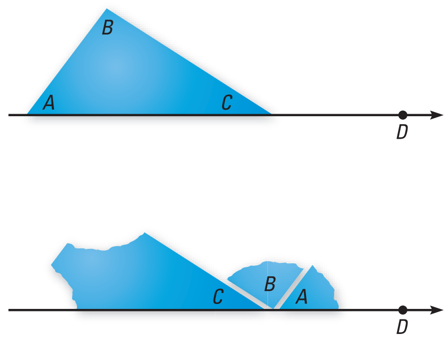
∠1, ∠2, and ∠3 are *interior angles*.

EXPLORE 2 **Find the measure of an exterior angle of a triangle**

STEP 1 ***Draw exterior angle*** Draw and cut out several different triangles. Place each triangle on a piece of paper and extend one side to form an *exterior angle*, as shown in the diagram.

STEP 2 ***Tear off corners*** For each triangle, tear off the corners that are not next to the exterior angle. Use them to fill the exterior angle, as shown.

STEP 3 ***Make a conjecture*** Make a conjecture about the relationship between the measure of an exterior angle of a triangle and the measures of the nonadjacent interior angles.



In the top figure, ∠*BCD* is an *exterior angle*.

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Given the measures of two interior angles of a triangle, how can you find the measure of the third angle? **Subtract the sum of the given measures from 180°.**
2. Draw several different triangles that each have one right angle. Show that the two acute angles of a right triangle are complementary. **Check students' work. The sum of the measures of the acute angles for each triangle should be 90°, so the angles are complementary.**

4.1 Apply Triangle Sum Properties

Before	You classified angles and found their measures.
Now	You will classify triangles and find measures of their angles.
Why?	So you can place actors on stage, as in Ex. 40.

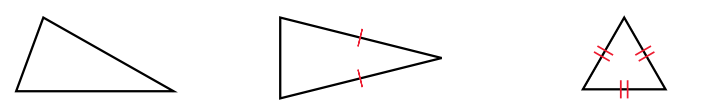
Key Vocabulary
- **triangle**
  scalene, isosceles, equilateral, acute, right, obtuse, equiangular
- **interior angles**
- **exterior angles**
- **corollary to a theorem**

A **triangle** is a polygon with three sides. A triangle with vertices A, B, and C is called "triangle ABC" or "$\triangle ABC$."

KEY CONCEPT *For Your Notebook*

Classifying Triangles by Sides

Scalene Triangle	Isosceles Triangle	Equilateral Triangle



No congruent sides	At least 2 congruent sides	3 congruent sides

Classifying Triangles by Angles

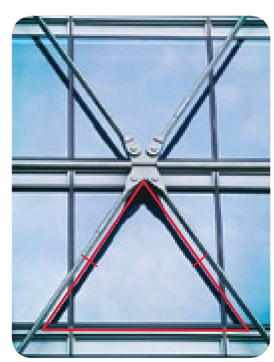
Acute Triangle	Right Triangle	Obtuse Triangle	Equiangular Triangle

3 acute angles	1 right angle	1 obtuse angle	3 congruent angles

READ VOCABULARY
Notice that an equilateral triangle is also isosceles. An equiangular triangle is also acute.

EXAMPLE 1 **Classify triangles by sides and by angles**

SUPPORT BEAMS Classify the triangular shape of the support beams in the diagram by its sides and by measuring its angles.



Solution

The triangle has a pair of congruent sides, so it is isosceles. By measuring, the angles are 55°, 55°, and 70°. It is an acute isosceles triangle.

(t), Nancy Sheehan/PhotoEdit, Inc.; (b), image 100/©Royalty-Free/Corbis

1 PLAN AND PREPARE

Warm-Up Exercises

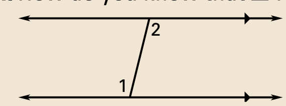
Also available online

Classify each angle as acute, obtuse, or right.

1. 90° **right** **2.** 72° **acute**

3. 116° **obtuse**

4. How do you know that $\angle 1 \cong \angle 2$?



Alt. Int. $\angle$s Thm.

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 1 day
Average: 1 day
Advanced: 1 day
Block: 0.5 block with next lesson
• See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 1

How can you find the measure of the third angle of a triangle if you know the measure of the other two angles? **Tell students they will learn how to answer this question by studying the Triangle Sum Theorem and its Corollary.**

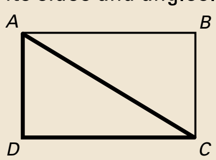
Motivating the Lesson

Ask students to think of pictures they have seen of the pyramids in Egypt. Ask them to imagine that they need to find the measure of the angle at the top of one of the triangular sides of a pyramid. Tell them that in this lesson, they will learn how to find the measure of the top angle by measuring the two angles at the bottom.

3 TEACH

Extra Example 1

Classify the triangle in the gate shown in the diagram by measuring its sides and angles.



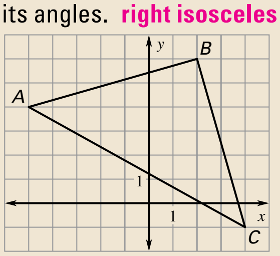
scalene triangle, right triangle

Key Question to Ask for Example 1

- Are there other kinds of triangles in the diagram? If so, classify them. **scalene right triangle, obtuse isosceles triangle**

Extra Example 2

Classify $\triangle ABC$ by its sides and by its angles. **right isosceles triangle**

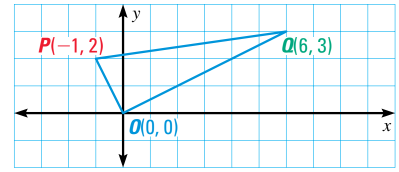


Key Question to Ask for Example 2

- How would you show $\triangle PQO$ is a right triangle using the lengths of the sides? **Note that $(\sqrt{5})^2 + (\sqrt{45})^2 = (\sqrt{50})^2$.**

EXAMPLE 2 **Classify a triangle in a coordinate plane**

Classify $\triangle PQO$ by its sides. Then determine if the triangle is a right triangle.



Solution

STEP 1 **Use** the distance formula to find the side lengths.

$$OP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((-1) - 0)^2 + (2 - 0)^2} = \sqrt{5} \approx 2.2$$

$$OQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - 0)^2 + (3 - 0)^2} = \sqrt{45} \approx 6.7$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - (-1))^2 + (3 - 2)^2} = \sqrt{50} \approx 7.1$$

STEP 2 **Check** for right angles. The slope of $\overline{OP}$ is $\frac{2-0}{-1-0} = -2$. The slope of $\overline{OQ}$ is $\frac{3-0}{6-0} = \frac{1}{2}$. The product of the slopes is $-2\left(\frac{1}{2}\right) = -1$, so $\overline{OP} \perp \overline{OQ}$ and $\angle POQ$ is a right angle.

▸ Therefore, $\triangle PQO$ is a right scalene triangle.

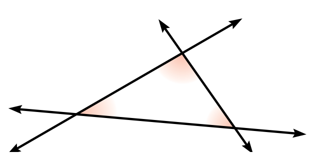
✓ **GUIDED PRACTICE** for Examples 1 and 2

1. Draw an obtuse isosceles triangle and an acute scalene triangle. **See margin.**

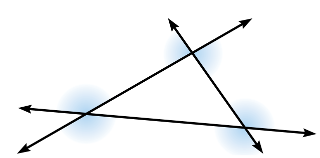
2. Triangle ABC has the vertices $A(0, 0)$, $B(3, 3)$, and $C(-3, 3)$. Classify it by its sides. Then determine if it is a right triangle. **isosceles; right triangle**

ANGLES When the sides of a polygon are extended, other angles are formed. The original angles are the **interior angles**. The angles that form linear pairs with the interior angles are the **exterior angles**.

READ DIAGRAMS
Each vertex has a *pair* of congruent exterior angles. However, it is common to show only *one* exterior angle at each vertex.



interior angles

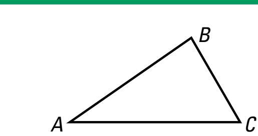


exterior angles

THEOREM *For Your Notebook*

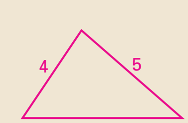
THEOREM 4.1 Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.



$m\angle A + m\angle B + m\angle C = 180°$

1. *Sample:*

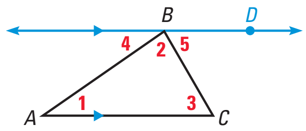


AUXILIARY LINES To prove certain theorems, you may need to add a line, a segment, or a ray to a given diagram. An *auxiliary* line is used in the proof of the Triangle Sum Theorem.

PROOF Triangle Sum Theorem

GIVEN ▶ $\triangle ABC$

PROVE ▶ $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Plan for Proof

a. Draw an auxiliary line through B and parallel to $\overline{AC}$.

b. Show that $m\angle 4 + m\angle 2 + m\angle 5 = 180°$, $\angle 1 \cong \angle 4$, and $\angle 3 \cong \angle 5$.

c. By substitution, $m\angle 1 + m\angle 2 + m\angle 3 = 180°$.

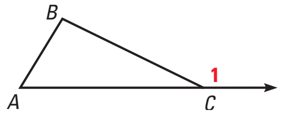
Plan in Action

	STATEMENTS	REASONS
a.	1. Draw $\overleftrightarrow{BD}$ parallel to $\overline{AC}$.	1. Parallel Postulate
b.	2. $m\angle 4 + m\angle 2 + m\angle 5 = 180°$	2. Angle Addition Postulate and definition of straight angle
	3. $\angle 1 \cong \angle 4$, $\angle 3 \cong \angle 5$	3. Alternate Interior Angles Theorem
	4. $m\angle 1 = m\angle 4$, $m\angle 3 = m\angle 5$	4. Definition of congruent angles
c.	5. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	5. Substitution Property of Equality

THEOREM *For Your Notebook*

THEOREM 4.2 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

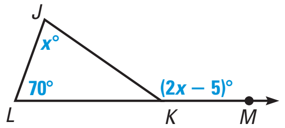


$m\angle 1 = m\angle A + m\angle B$

INEQUALITIES Theorem 4.2 implies that the following inequalities are true:
$m\angle 1 > m\angle A$
$m\angle 1 > m\angle B$

EXAMPLE 3 Find an angle measure

xy ALGEBRA **Find $m\angle JKM$.**



Solution

STEP 1 **Write** and solve an equation to find the value of x.

$(2x - 5)° = 70° + x°$ **Apply the Exterior Angle Theorem.**

$x = 75$ **Solve for x.**

STEP 2 **Substitute** 75 for x in $2x - 5$ to find $m\angle JKM$.

$2x - 5 = 2 \cdot 75 - 5 = 145$

▶ The measure of $\angle JKM$ is 145°.

Teaching Strategy

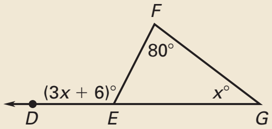
Review the distance formula and slope formula. Remind students that they can use the distance formula to determine whether or not sides are congruent. To decide whether an angle is a right angle, they can use slopes. Slopes must be negative reciprocals for two nonvertical sides to be perpendicular.

Avoiding Common Errors

Example 3 Students may find x but forget to substitute to find $m\angle JKM$. Tell them to look back at the original problem to be sure they have found what was asked for.

Extra Example 3

Find $m\angle DEF$. **117°**



Key Questions to Ask for Example 3

- How can you use the fact that $m\angle JKM = (2x - 5)°$ to write an expression for $m\angle JKL$? **$\angle JKM$ and $\angle JKL$ are supplementary, so $m\angle JKL = 180° - (2x - 5)°$.**
- What expression do you get for $m\angle JKL$ if you use the Triangle Sum Theorem? **$180° - (x° + 70°)$**
- How can you use these expressions to find x? **Solve $180 - (2x - 5) = 180 - (x + 70)$.**

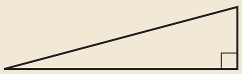
Differentiated Instruction

Inclusion To help students think about why the Exterior Angle Theorem is true in **Example 3**, have them review the Angle Addition Postulate and the definition of a straight angle. Have students write an equation to find the value of the third interior angle of the triangle: $180° - (2x - 5)° = m\angle 3$. Then have them use the Triangle Sum Theorem to write another equation to find the value of the third interior angle: $180° - (70 + x)° = m\angle 3$. When comparing the two equations, students should notice that $(2x - 5)° = (70 + x)°$.

See also the *Differentiated Instruction Resources* for more strategies.

Extra Example 4

The support for the skateboard ramp shown forms a right triangle. The measure of one acute angle in the triangle is five times the measure of the other. Find the measure of each acute angle. **15°, 75°**



Key Question to Ask for Example 4

- If you use the Triangle Sum Theorem to solve this problem, what equation would you write? What can you do to this equation to get the equation obtained from the corollary? **$x + 2x + 90 = 180$; Subtract 90 from both sides.**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How can you find the measure of the third angle of a triangle if you know the measures of the other two angles?

- **Equilateral triangles have three congruent sides, isosceles triangles have at least two congruent sides, and scalene triangles have no congruent sides.**
- **Equiangular triangles have three congruent angles, acute triangles have three acute angles, obtuse triangles have one obtuse angle, and right triangles have one right angle.**
- **The sum of the measures of the interior angles of a triangle is 180°.**

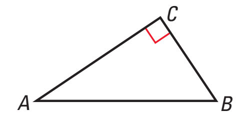
Add the two known angle measures and subtract the result from 180°.

A ______ is a statement that can be proved easily using the theorem. The corollary below follows from the Triangle Sum Theorem.

COROLLARY *For Your Notebook*

Corollary to the Triangle Sum Theorem

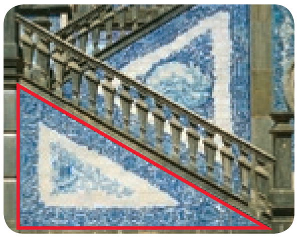
The acute angles of a right triangle are complementary.



$m\angle A + m\angle B = 90°$

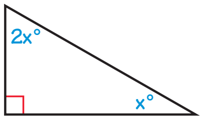
EXAMPLE 4 Find angle measures from a verbal description

ARCHITECTURE The tiled staircase shown forms a right triangle. The measure of one acute angle in the triangle is twice the measure of the other. Find the measure of each acute angle.



Solution

First, sketch a diagram of the situation. Let the measure of the smaller acute angle be $x°$. Then the measure of the larger acute angle is $2x°$. The Corollary to the Triangle Sum Theorem states that the acute angles of a right triangle are complementary.



Use the corollary to set up and solve an equation.

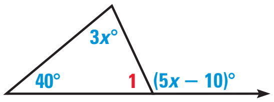
$x° + 2x° = 90°$ **Corollary to the Triangle Sum Theorem**

$x = 30$ **Solve for x.**

▶ So, the measures of the acute angles are 30° and $2(30°) = 60°$.

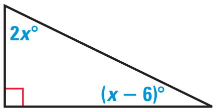
✓ GUIDED PRACTICE for Examples 3 and 4

3. Find the measure of $\angle 1$ in the diagram shown. **65°**



4. Find the measure of each interior angle of $\triangle ABC$, where $m\angle A = x°$, $m\angle B = 2x°$, and $m\angle C = 3x°$. **$m\angle A = 30°$, $m\angle B = 60°$, $m\angle C = 90°$**

5. Find the measures of the acute angles of the right triangle in the diagram shown. **26°, 64°**



6. In Example 4, what is the measure of the obtuse angle formed between the staircase and a segment extending from the horizontal leg? **150°**

Nicolas Sapieha/Corbis

4.1 EXERCISES

HOMEWORK KEY
○ = See **WORKED-OUT SOLUTIONS** Exs. 9, 15, and 41
★ = **STANDARDIZED TEST PRACTICE** Exs. 7, 20, 31, 43, and 51

SKILL PRACTICE

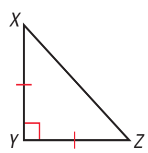
A **VOCABULARY** **Match the triangle description with the most specific name.**

1. Angle measures: 30°, 60°, 90° **C**	**A.** Isosceles
2. Side lengths: 2 cm, 2 cm, 2 cm **E**	**B.** Scalene
3. Angle measures: 60°, 60°, 60° **F**	**C.** Right
4. Side lengths: 6 m, 3 m, 6 m **A**	**D.** Obtuse
5. Side lengths: 5 ft, 7 ft, 9 ft **B**	**E.** Equilateral
6. Angle measures: 20°, 125°, 35° **D**	**F.** Equiangular

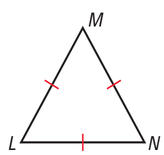
7. ★ **WRITING** Can a right triangle also be obtuse? *Explain* why or why not.
No; in a right triangle, the other two angles are complementary so they are both less than 90°.

EXAMPLE 1 for Exs. 8–10

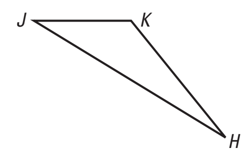
CLASSIFYING TRIANGLES **Copy the triangle and measure its angles. Classify the triangle by its sides and by its angles.**

8.


isosceles, right

9.


equilateral, equiangular

10.


scalene, obtuse

EXAMPLE 2 for Exs. 11–13

COORDINATE PLANE **A triangle has the given vertices. Graph the triangle and classify it by its sides. Then determine if it is a right triangle.** **11–13. See margin for art.**

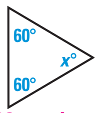
11. $A(2, 3)$, $B(6, 3)$, $C(2, 7)$ **isosceles; right triangle**

12. $A(3, 3)$, $B(6, 9)$, $C(6, -3)$ **isosceles; not a right triangle**

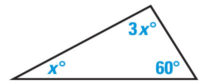
13. $A(1, 9)$, $B(4, 8)$, $C(2, 5)$ **scalene; not a right triangle**

EXAMPLE 3 for Exs. 14–19

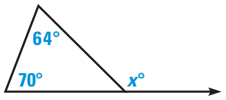
FINDING ANGLE MEASURES **Find the value of x. Then classify the triangle by its angles.**

14.


60; equiangular

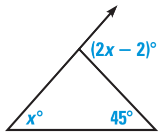
15.


30; right

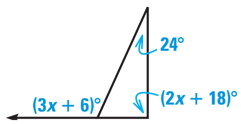
16.


134; acute

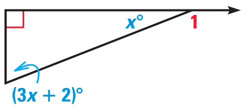
xy ALGEBRA **Find the measure of the exterior angle shown.**

17.


92°

18.


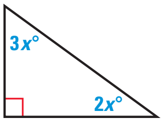
114°

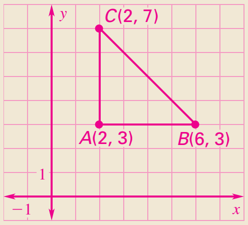
19.


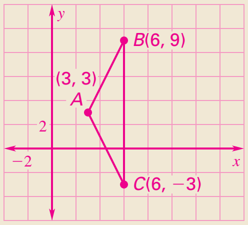
158°

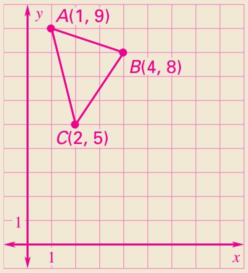
EXAMPLE 4 for Ex. 20

20. ★ **SHORT RESPONSE** *Explain* how to use the Corollary to the Triangle Sum Theorem to find the measure of each angle.
Set $3x + 2x = 90$ and solve for x. Then find the values of $3x$ and $2x$.



11.


12.


13.


4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1: EP for 4.1 Exs. 24–29
Exs. 1–7, 9–19 odd, 21–29, 40–49

Average:
Day 1:
Exs. 1–7, 8–26 even, 27–34, 40–52

Advanced:
Day 1:
Exs. 1–7, 10, 13, 16, 19, 20, 27, 28, 31–40*, 42–53*

Block:
Exs. 1–7, 8–26 even, 27–34, 40–52 (with next lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 9, 11, 17, 19, 40
Average: 8, 12, 18, 20, 40
Advanced: 10, 13, 19, 20, 40

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

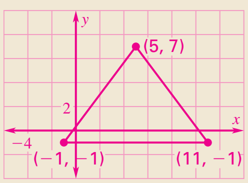
Avoiding Common Errors

Exercises 17–18 Students may add the measure of the exterior angle and the measures of the two nonadjacent interior angles and set the sum equal to 180°. Discuss the difference between the Triangle Sum Theorem and the Exterior Angle Theorem to help them understand why that is not a correct procedure.

Study Strategy

Exercises 32–33, 35 If students have difficulty with these exercises, call attention to the fact that each diagram is marked to show a pair of parallel segments.

39c.



29. Isosceles does not guarantee the third side is congruent to the two congruent sides; so if △*ABC* is equilateral, then it is isosceles as well.

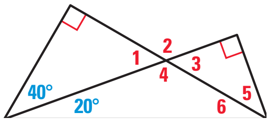
30. The measure of the exterior angle is equal to the sum of the measures of the two nonadjacent interior angles; $m\angle 1 = 80° + 50° = 130°$.

38. No. *Sample answer:* In a right triangle, the two acute angles are complementary. So, one of the acute angle measures can be as small as desired, while the other angle measure is less than 90°. The largest angle is the right angle, which measures 90°, so the triangle does not need to be obtuse.

39a. *Sample answer:* They will always form a triangle unless they intersect in one point, or unless at least two lines are parallel.

B **ANGLE RELATIONSHIPS** **Find the measure of the numbered angle.**

21. ∠1 **50°**
22. ∠2 **130°**
23. ∠3 **50°**
24. ∠4 **130°**
25. ∠5 **40°**
26. ∠6 **30°**



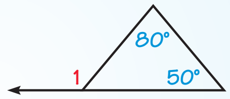
27. **ALGEBRA** In △*PQR*, ∠*P* ≅ ∠*R* and the measure of ∠*Q* is twice the measure of ∠*R*. Find the measure of each angle. **$m\angle P = 45°$, $m\angle Q = 90°$, $m\angle R = 45°$**

28. **ALGEBRA** In △*EFG*, $m\angle F = 3(m\angle G)$, and $m\angle E = m\angle F - 30°$. Find the measure of each angle. **$m\angle E = 60°$, $m\angle F = 90°$, $m\angle G = 30°$**

ERROR ANALYSIS **In Exercises 29 and 30, *describe* and correct the error.**

29. All equilateral triangles are also isosceles. So, if △ABC is isosceles, then it is equilateral as well.

30. m∠1 + 80° + 50° = 180°

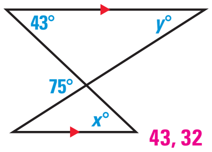


31. ★ **MULTIPLE CHOICE** Which of the following is not possible? **B**

Ⓐ An acute scalene triangle
Ⓑ A triangle with two acute exterior angles
Ⓒ An obtuse isosceles triangle
Ⓓ An equiangular acute triangle

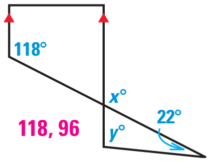
ALGEBRA **In Exercises 32–37, find the values of *x* and *y*.**

32.



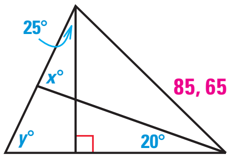
43, 32

33.



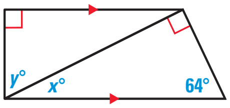
118, 96

34.



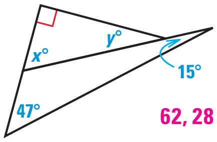
85, 65

35.



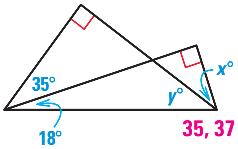
26, 64

36.



62, 28

37.



35, 37

C

38. **VISUALIZATION** Is there an angle measure that is so small that any triangle with that angle measure will be an obtuse triangle? *Explain.*

39. **CHALLENGE** Suppose you have the equations $y = ax + b$, $y = cx + d$, and $y = ex + f$.

a. When will these three lines form a triangle?

b. Let $c = 1$, $d = 2$, $e = 4$, and $f = -7$. Find values of *a* and *b* so that no triangle is formed by the three equations. ***Sample answer:* 0, 5**

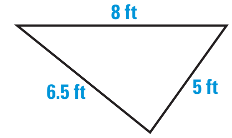
c. Draw the triangle formed when $a = \frac{4}{3}$, $b = \frac{1}{3}$, $c = -\frac{4}{3}$, $d = \frac{41}{3}$, $e = 0$, and $f = -1$. Then classify the triangle by its sides. **See margin for art; isosceles.**

◯ = See **WORKED-OUT SOLUTIONS** in Student Resources   ★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

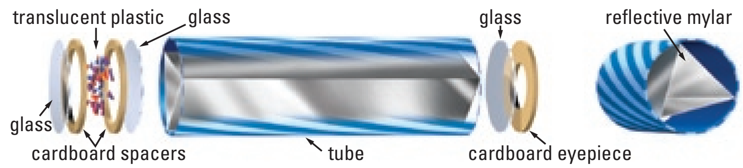
EXAMPLE 1 A
for Ex. 40

40. THEATER Three people are standing on a stage. The distances between the three people are shown in the diagram. Classify the triangle formed by its sides. Then copy the triangle, measure the angles, and classify the triangle by its angles. **scalene; acute**



41. 2 in.; 60°; in an equilateral triangle all sides have the same length $\left(\frac{6}{3}\right)$. In an equiangular triangle the angles always measure 60°.

41. KALEIDOSCOPES You are making a kaleidoscope. The directions state that you are to arrange three pieces of reflective mylar in an equilateral and equiangular triangle. You must cut three strips from a piece of mylar 6 inches wide. What are the side lengths of the triangle used to form the kaleidoscope? What are the measures of the angles? *Explain.*

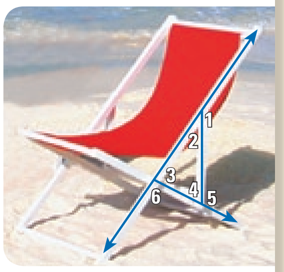


42. SCULPTURE You are bending a strip of metal into an isosceles triangle for a sculpture. The strip of metal is 20 inches long. The first bend is made 6 inches from one end. *Describe* two ways you could complete the triangle. **Bend the strip again at 7 inches or 8 inches from the other end.**

43. ★ MULTIPLE CHOICE Which inequality describes the possible measures of an angle of a triangle? **C**

(A) $0° \le x° \le 180°$ (B) $0° \le x° < 180°$ (C) $0° < x° < 180°$ (D) $0° < x° \le 180°$

SLING CHAIRS The brace of a sling chair forms a triangle with the seat and legs of the chair. Suppose $m\angle 2 = 50°$ and $m\angle 3 = 65°$.



44. Find $m\angle 6$. **115°**

45. Find $m\angle 5$. **115°**

46. Find $m\angle 1$. **130°**

47. Find $m\angle 4$. **65°**

B

48. PROOF Prove the Corollary to the Triangle Sum Theorem. **See margin.**

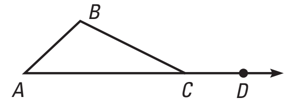
49. MULTI-STEP PROBLEM The measures of the angles of a triangle are $(2\sqrt{2x})°$, $(5\sqrt{2x})°$, and $(2\sqrt{2x})°$.

a. Write an equation to show the relationship of the angles. $2\sqrt{2x} + 5\sqrt{2x} + 2\sqrt{2x} = 180$

b. Find the measure of each angle. **40°, 100°, 40°**

c. Classify the triangle by its angles. **obtuse**

50. PROVING THEOREM 4.2 Prove the Exterior Angle Theorem. (*Hint:* Find two equations involving $m\angle ACB$.) **See margin.**



Mathematical Reasoning

Exercise 42 Have students relate the two ways of completing the triangle to the definition of an isosceles triangle.

48. Statements (Reasons)

1. $\triangle ABC$ is a right triangle. (Given)
2. $m\angle C = 90°$ (Definition of right angle)
3. $m\angle A + m\angle B + m\angle C = 180°$ (Triangle Sum Theorem)
4. $m\angle A + m\angle B + 90° = 180°$ (Substitution Property of Equality)
5. $m\angle A + m\angle B = 90°$ (Subtraction Property of Equality)
6. $\angle A$ and $\angle B$ are complementary. (Definition of complementary angles)

50. Statements (Reasons)

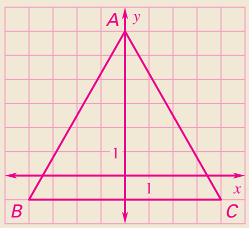
1. $m\angle ACB + m\angle BCD = 180°$ (Linear Pair Postulate and definition of supplementary angles)
2. $m\angle A + m\angle B + m\angle ACB = 180°$ (Triangle Sum Theorem)
3. $m\angle ACB + m\angle BCD = m\angle A + m\angle B + m\angle ACB$ (Transitive Property of Equality)
4. $m\angle BCD = m\angle A + m\angle B$ (Subtraction Property of Equality)

5 ASSESS AND RETEACH

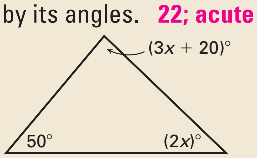
Daily Homework Quiz

Also available online

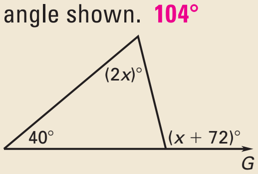
1. Graph $\triangle ABC$ with vertices $A(0, 6)$, $B(-4, -1)$, and $C(4, -1)$. Classify it by its sides. Then determine if it is a right triangle. **isosceles; not a right triangle**



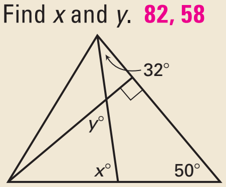
2. Find x. Then classify the triangle by its angles. **22; acute**



3. Find the measure of the exterior angle shown. **104°**



4. Find x and y. **82, 58**



Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

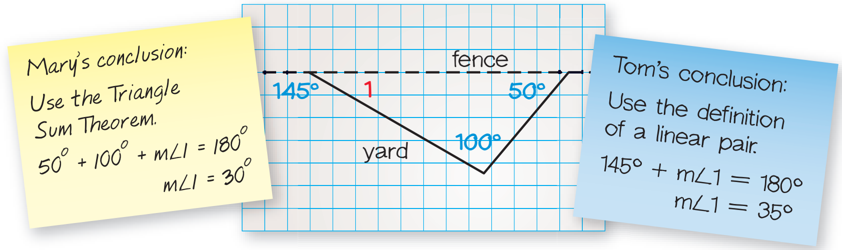
Challenge

Additional challenge is available in the Chapter Resource Book

53. See Additional Answers.

51. *Sample answer:* They both reasoned correctly but their initial plan was incorrect. The measure of the exterior angle should be 150°.

51. ★ **EXTENDED RESPONSE** The figure below shows an initial plan for a triangular flower bed that Mary and Tom plan to build along a fence. They are discussing what the measure of $\angle 1$ should be.



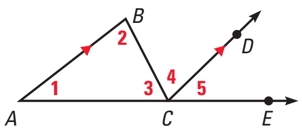
Did Mary and Tom both reason correctly? If not, who made a mistake and what mistake was made? If they did both reason correctly, what can you conclude about their initial plan? *Explain.*

52. **xy ALGEBRA** $\triangle ABC$ is isosceles. $AB = x$ and $BC = 2x - 4$.

a. Find two possible values for x if the perimeter of $\triangle ABC$ is 32. **8, 9**

b. How many possible values are there for x if the perimeter of $\triangle ABC$ is 12? **one value**

C **53.** **CHALLENGE** Use the diagram to write a proof of the Triangle Sum Theorem. Your proof should be different than the proof of the Triangle Sum Theorem shown in this lesson. **See margin.**



See **EXTRA PRACTICE** in Student Resources **ONLINE QUIZ** at my.hrw.com

4.2 Apply Congruence and Triangles

Before You identified congruent angles.

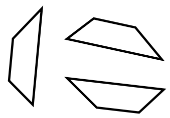
Now You will identify congruent figures.

Why? So you can determine if shapes are identical, as in Example 3.

Key Vocabulary
- **congruent figures**
- **corresponding parts**

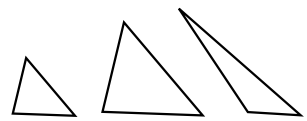
Two geometric figures are *congruent* if they have exactly the same size and shape. Imagine cutting out one of the congruent figures. You could then position the cut-out figure so that it fits perfectly onto the other figure.

Congruent



Same size and shape

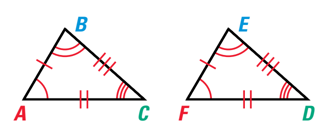
Not congruent



Different sizes or shapes

In two **congruent figures**, all the parts of one figure are congruent to the **corresponding parts** of the other figure. In congruent polygons, this means that the *corresponding sides* and the *corresponding angles* are congruent.

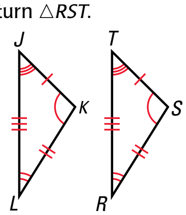
CONGRUENCE STATEMENTS When you write a congruence statement for two polygons, always list the corresponding vertices in the same order. You can write congruence statements in more than one way. Two possible congruence statements for the triangles at the right are $\triangle ABC \cong \triangle FED$ or $\triangle BCA \cong \triangle EDF$.



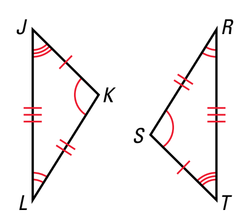
Corresponding angles	$\angle A \cong \angle F$	$\angle B \cong \angle E$	$\angle C \cong \angle D$
Corresponding sides	$\overline{AB} \cong \overline{FE}$	$\overline{BC} \cong \overline{ED}$	$\overline{AC} \cong \overline{FD}$

EXAMPLE 1 Identify congruent parts

VISUAL REASONING
To help you identify corresponding parts, turn $\triangle RST$.



Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.



Solution

The diagram indicates that $\triangle JKL \cong \triangle TSR$.

Corresponding angles $\angle J \cong \angle T$, $\angle K \cong \angle S$, $\angle L \cong \angle R$

Corresponding sides $\overline{JK} \cong \overline{TS}$, $\overline{KL} \cong \overline{SR}$, $\overline{LJ} \cong \overline{RT}$

Image Source/Alamy

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

1. When are two angles congruent? **when they have the same measure**

2. In $\triangle ABC$, if $m\angle A = 64°$ and $m\angle B = 71°$, what is $m\angle C$? **45°**

3. What property of angle congruence is illustrated by this statement? If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$. **Transitive Property**

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days
Average: 2 days
Advanced: 2 days
Block: 0.5 block with previous lesson
0.5 block with next lesson
- See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 2

What are congruent figures? **Tell students they will learn how to answer this question by studying the definition of congruent figures.**

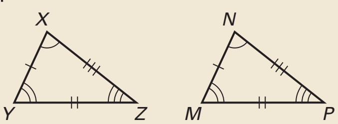
Motivating the Lesson

To replace missing or damaged tiles in old houses, it is often necessary to supply information to get tiles made to order. Tell students that in this lesson, they will learn what kinds of information they might need to supply to ensure they get tiles that are just the size and shape required.

3 TEACH

Extra Example 1

Write a congruence statement for the triangles shown. Identify all pairs of congruent corresponding parts.



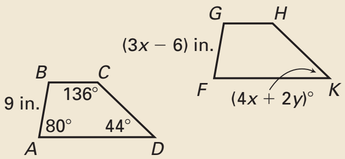
$\triangle XYZ \cong \triangle NMP$, $\overline{XY} \cong \overline{NM}$, $\overline{XZ} \cong \overline{NP}$, $\overline{YZ} \cong \overline{MP}$, $\angle X \cong \angle N$, $\angle Y \cong \angle M$, $\angle Z \cong \angle P$

Key Question to Ask for Example 1

- Is there any other way you could have written the congruence statement? Explain. **Yes; another statement is $\triangle KLJ \cong \triangle SRT$.**

Extra Example 2

In the diagram, $ABCD \cong FGHK$.



a. Find the value of x. **5**

b. Find the value of y. **12**

Key Question to Ask for Example 2

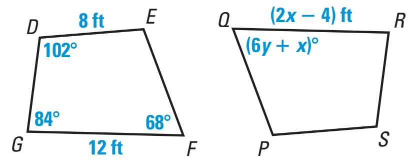
- What are the measures of $\angle S$ and $\angle R$? **102°, 84°**

EXAMPLE 2 Use properties of congruent figures

In the diagram, $DEFG \cong SPQR$.

a. Find the value of x.

b. Find the value of y.



Solution

a. You know that $\overline{FG} \cong \overline{QR}$.

$FG = QR$

$12 = 2x - 4$

$16 = 2x$

$8 = x$

b. You know that $\angle F \cong \angle Q$.

$m\angle F = m\angle Q$

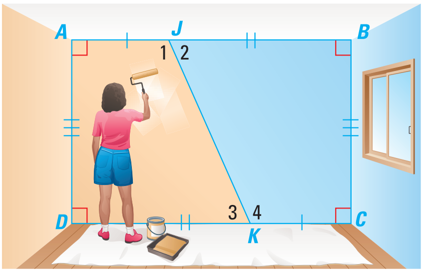
$68° = (6y + x)°$

$68 = 6y + 8$

$10 = y$

EXAMPLE 3 Show that figures are congruent

PAINTING If you divide the wall into orange and blue sections along $\overline{JK}$, will the sections of the wall be the same size and shape? *Explain.*



Solution

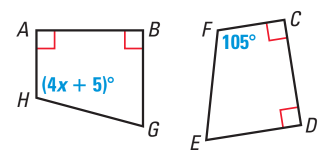
From the diagram, $\angle A \cong \angle C$ and $\angle D \cong \angle B$ because all right angles are congruent. Also, by the Lines Perpendicular to a Transversal Theorem, $\overline{AB} \parallel \overline{DC}$. Then, $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$ by the Alternate Interior Angles Theorem. So, all pairs of corresponding angles are congruent.

The diagram shows $\overline{AJ} \cong \overline{CK}$, $\overline{KD} \cong \overline{JB}$, and $\overline{DA} \cong \overline{BC}$. By the Reflexive Property, $\overline{JK} \cong \overline{KJ}$. All corresponding parts are congruent, so $AJKD \cong CKJB$.

▶ Yes, the two sections will be the same size and shape.

✓ GUIDED PRACTICE for Examples 1, 2, and 3

In the diagram at the right, $ABGH \cong CDEF$.



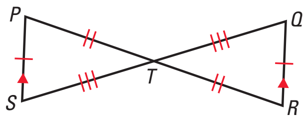
1. Identify all pairs of congruent corresponding parts.

1. $\overline{AB} \cong \overline{CD}$, $\overline{BG} \cong \overline{DE}$, $\overline{GH} \cong \overline{EF}$, $\overline{HA} \cong \overline{FC}$, $\angle A \cong \angle C$, $\angle B \cong \angle D$, $\angle G \cong \angle E$, $\angle H \cong \angle F$

2. Find the value of x and find $m\angle H$. **25, 105°**

3. Show that $\triangle PTS \cong \triangle RTQ$.

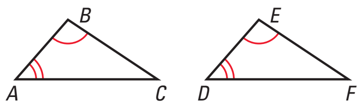
All of the corresponding parts of $\triangle PTS$ are congruent to those of $\triangle RTQ$ by the indicated markings, the Vertical Angles Theorem, and the Alternate Interior Angles Theorem.



THEOREM *For Your Notebook*

THEOREM 4.3 Third Angles Theorem

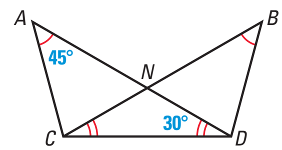
If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.



If $\angle A \cong \angle D$, and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.

EXAMPLE 4 Use the Third Angles Theorem

Find $m\angle BDC$.



Solution

$\angle A \cong \angle B$ and $\angle ADC \cong \angle BCD$, so by the Third Angles Theorem, $\angle ACD \cong \angle BDC$. By the Triangle Sum Theorem, $m\angle ACD = 180° - 45° - 30° = 105°$.

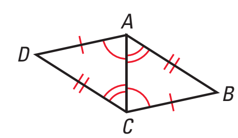
▶ So, $m\angle ACD = m\angle BDC = 105°$ by the definition of congruent angles.

ANOTHER WAY

For an alternative method for solving the problem in Example 4, turn for the **Problem Solving Workshop**.

EXAMPLE 5 Prove that triangles are congruent

Write a proof.



GIVEN ▶ $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$, $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$

PROVE ▶ $\triangle ACD \cong \triangle CAB$

Plan for Proof
a. Use the Reflexive Property to show that $\overline{AC} \cong \overline{AC}$.
b. Use the Third Angles Theorem to show that $\angle B \cong \angle D$.

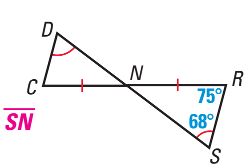
Plan in Action

	STATEMENTS	REASONS
	1. $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$	1. Given
a.	2. $\overline{AC} \cong \overline{AC}$	2. Reflexive Property of Congruence
	3. $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$	3. Given
b.	4. $\angle B \cong \angle D$	4. Third Angles Theorem
	5. $\triangle ACD \cong \triangle CAB$	5. Definition of $\cong$ figures

✓ GUIDED PRACTICE for Examples 4 and 5

4. In the diagram, what is $m\angle DCN$? **75°**

5. By the definition of congruence, what additional information is needed to know that $\triangle NDC \cong \triangle NSR$? **$\overline{DC} \cong \overline{SR}$ and $\overline{DN} \cong \overline{SN}$**



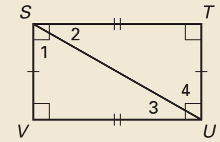
Differentiated Instruction

Kinesthetic Learners Some students may find it challenging to visualize that two triangles in different orientations are congruent. Have these students trace the triangles in **Example 5** onto tracing paper, cut them out, and arrange them so one triangle fits exactly on top of the other.

See also the *Differentiated Instruction Resources* for more strategies.

Extra Example 3

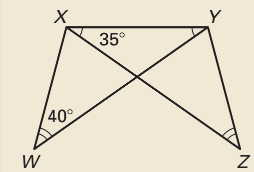
Maggie took the piece of fabric *STUV* shown in the diagram and cut it on the diagonal to make a scarf for her and a friend. Are the two pieces the same size and shape? Explain.



Yes; $\angle V \cong \angle VUT \cong \angle T$ since all are right angles. $\overline{ST} \parallel \overline{VU}$ and $\overline{SV} \parallel \overline{TU}$ by the Lines Perpendicular to a Transversal Theorem. $\angle 2 \cong \angle 3$ and $\angle 1 \cong \angle 4$ by the Alternate Interior Angles Theorem. It is given that $\overline{SV} \cong \overline{UT}$ and $\overline{VU} \cong \overline{TS}$, and $\overline{SU} \cong \overline{SU}$ by the Refl. Prop. of $\cong$. All corresponding parts are $\cong$, so $\triangle UVS \cong \triangle STU$.

Extra Example 4

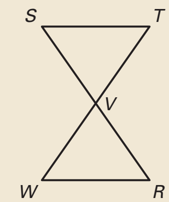
Find $m\angle YXW$. **105°**



Extra Example 5

Given: $\overline{SV} \cong \overline{RV}$, $\overline{TV} \cong \overline{WV}$, $\overline{ST} \cong \overline{RW}$, $\angle T \cong \angle W$

Prove: $\triangle STV \cong \triangle RWV$



Statements (Reasons)

1. $\overline{SV} \cong \overline{RV}$, $\overline{TV} \cong \overline{WV}$, $\overline{ST} \cong \overline{RW}$ (Given)
2. $\angle T \cong \angle W$ (Given)
3. $\angle SVT \cong \angle RVW$ (Vert. $\angle$s Thm.)
4. $\angle S \cong \angle R$ (Third $\angle$s Thm.)
5. $\triangle STV \cong \triangle RWV$ (Def. of $\cong$ $\triangle$s)

Key Question to Ask for Example 5

- How many pairs of sides and pairs of angles must you show are congruent when you use the definition of congruent triangles to show that two triangles are congruent? **three pairs of sides, three pairs of angles**

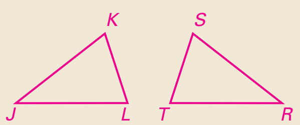
Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: What are congruent figures?

- **Triangles can be proved congruent by showing that all 3 pairs of corresponding sides and all 3 pairs of corresponding angles are congruent.**
- **If two angles of one triangle are congruent to two angles of another, then the third angles are congruent.**

Congruent figures are figures that have exactly the same size and shape.

1.



2. 3 pairs of congruent sides and 3 pairs of congruent angles; to prove two figures congruent it must be shown that all corresponding sides and angles are congruent.

3. $\angle A$ **and** $\angle D$, $\angle C$ **and** $\angle F$, $\angle B$ **and** $\angle E$, $\overline{AB}$ **and** $\overline{DE}$, $\overline{AC}$ **and** $\overline{DF}$, $\overline{BC}$ **and** $\overline{EF}$. ***Sample answer:*** $\triangle CAB \cong \triangle FDE$

4. $\angle G$ **and** $\angle Q$, $\angle H$ **and** $\angle R$, $\angle K$ **and** $\angle T$, $\angle J$ **and** $\angle S$, $\overline{GH}$ **and** $\overline{QR}$, $\overline{HJ}$ **and** $\overline{RS}$, $\overline{JK}$ **and** $\overline{ST}$, $\overline{KG}$ **and** $\overline{TQ}$. ***Sample answer:*** $HJKG \cong RSTQ$

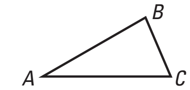
PROPERTIES OF CONGRUENT TRIANGLES The properties of congruence that are true for segments and angles are also true for triangles.

THEOREM *For Your Notebook*

THEOREM 4.4 Properties of Congruent Triangles

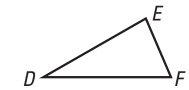
Reflexive Property of Congruent Triangles

For any triangle ABC, $\triangle ABC \cong \triangle ABC$.



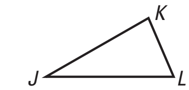
Symmetric Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.



Transitive Property of Congruent Triangles

If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.



4.2 EXERCISES

HOMEWORK KEY

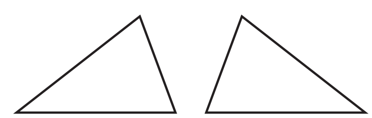
○ = See **WORKED-OUT SOLUTIONS** Exs. 9, 15, and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, 21, 24, 27, and 30

SKILL PRACTICE

A

1. **VOCABULARY** Copy the congruent triangles shown. Then label the vertices of the triangles so that $\triangle JKL \cong \triangle RST$. Identify all pairs of congruent *corresponding angles* and *corresponding sides*.



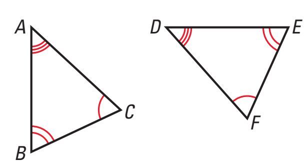
$\overline{JK} \cong \overline{RS}$, $\overline{KL} \cong \overline{ST}$, $\overline{JL} \cong \overline{RT}$, $\angle J \cong \angle R$, $\angle K \cong \angle S$, $\angle L \cong \angle T$; see margin for art.

2. ★ **WRITING** Based on this lesson, what information do you need to prove that two triangles are congruent? *Explain*. **See margin.**

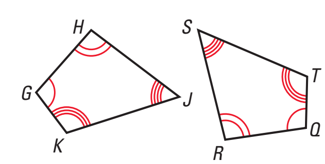
EXAMPLE 1 for Exs. 3–4

USING CONGRUENCE Identify all pairs of congruent corresponding parts. Then write another congruence statement for the figures. 3, 4. See margin.

3. $\triangle ABC \cong \triangle DEF$



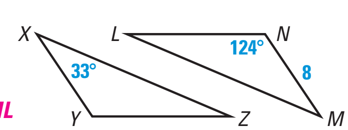
4. $GHJK \cong QRST$



EXAMPLE 2 for Exs. 5–10

READING A DIAGRAM In the diagram, $\triangle XYZ \cong \triangle MNL$. Copy and complete the statement.

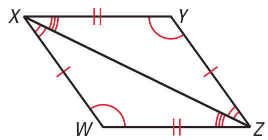
5. $m\angle Y =$ __?__ **124°**
6. $m\angle M =$ __?__ **33°**
7. $YX =$ __?__ **8**
8. $\overline{YZ} \cong$ __?__ $\overline{NL}$
9. $\triangle LNM \cong$ __?__ $\triangle ZYX$
10. $\triangle YXZ \cong$ __?__ $\triangle NML$



EXAMPLE 3
for Exs. 11–14

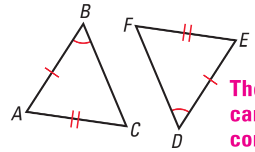
NAMING CONGRUENT FIGURES **Write a congruence statement for any figures that can be proved congruent. *Explain* your reasoning.**

11.



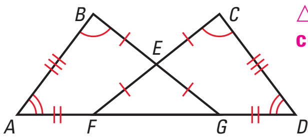
$\triangle XYZ \cong \triangle ZWX$; all corresponding sides and angles are congruent.

12.



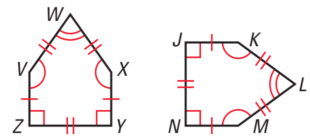
The triangles cannot be proved congruent.

13.



$\triangle BAG \cong \triangle CDF$; all corresponding sides and angles are congruent.

14.

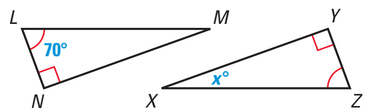


$VWXYZ \cong KLMNJ$ or $VWXYZ \cong MLKJN$; all corresponding sides and angles are congruent.

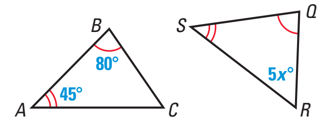
EXAMPLE 4
for Exs. 15–16

THIRD ANGLES THEOREM **Find the value of x.**

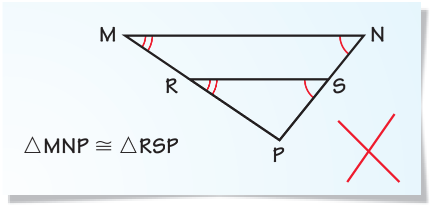
15. 20



16. 11



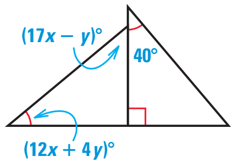
B **17. ERROR ANALYSIS** A student says that $\triangle MNP \cong \triangle RSP$ because the corresponding angles of the triangles are congruent. *Describe* the error in this statement. **Student still needs to show that corresponding sides are congruent.**



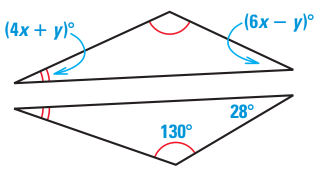
18. ★ OPEN-ENDED MATH Graph the triangle with vertices $L(3, 1)$, $M(8, 1)$, and $N(8, 8)$. Then graph a triangle congruent to $\triangle LMN$. **See margin.**

ALGEBRA **Find the values of x and y.**

19. 3, 1



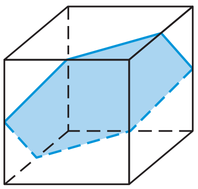
20. 5, 2



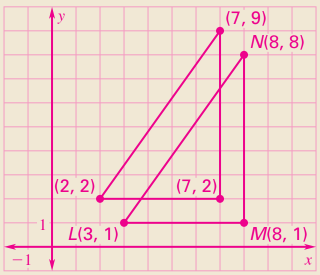
21. ★ MULTIPLE CHOICE Suppose $\triangle ABC \cong \triangle EFD$, $\triangle EFD \cong \triangle GIH$, $m\angle A = 90°$, and $m\angle F = 20°$. What is $m\angle H$? **B**

(A) 20° (B) 70° (C) 90° (D) Cannot be determined

C **22. CHALLENGE** A hexagon is contained in a cube, as shown. Each vertex of the hexagon lies on the midpoint of an edge of the cube. This hexagon is equiangular. *Explain* why it is also regular. **It is regular because all angles are congruent and all sides will be congruent because they are corresponding parts of congruent triangles.**



18. *Sample:*



4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–14
Day 2:
Exs. 15–19, 23–28

Average:
Day 1:
Exs. 1–14
Day 2:
Exs. 15–21, 23–31

Advanced:
Day 1:
Exs. 1–4, 7–14, 22*
Day 2:
Exs. 15–21, 24–32*

Block:
Exs. 1–14 (with previous lesson)
Exs. 15–21, 23–31(with next lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 6, 12, 16, 26
Average: 3, 8, 13, 16, 26
Advanced: 4, 10, 14, 16, 26

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

Mathematical Reasoning

Exercise 1 Discuss the number of ways the letters J, K, and L can be used to label the vertices of the triangle on the left. Assuming it has been decided how to label the vertices of that triangle, ask students if there is more than one correct way to label the vertices of the triangle on the right. Stress that only one correct labeling is possible.

Avoiding Common Errors

Exercises 11–14 Students may write the letters in the wrong order in the congruence statement. Review how the order of the letters in a congruence statement tells which sides and angles correspond to one another.

Mathematical Reasoning

Exercises 19–20 Ask students whether the triangles in each exercise are or are not congruent. Help them understand that despite the diagram, it is not possible to say whether the triangles are or are not congruent, since we have no definite information about the sides of the triangles.

Teaching Strategy

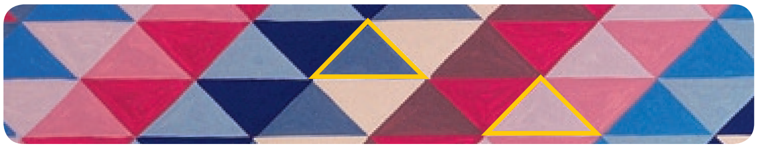
Exercise 27 If students have trouble drawing an appropriate figure, it may help them to first draw and label $\triangle ABC$. They can put tracing paper over the triangle, trace it, and label the vertices of the copy. They can then change the position of the copy so that points B and C of the copy fall on top of points C and B of the original triangle. This will enable them to determine an appropriate location for point D.

Avoiding Common Errors

Exercise 28 Remind students that since they are proving the Third Angle Theorem, they may not use the Third Angle Theorem as a reason in their proof. They should follow the Plan for the Proof that is given in the exercise.

Problem Solving

A **23. RUG DESIGNS** The rug design is made of congruent triangles. One triangular shape is used to make all of the triangles in the design. Which property guarantees that all the triangles are congruent? **Transitive Property of Congruent Triangles**



24. ★ **OPEN-ENDED MATH** Create a design for a rug made with congruent triangles that is different from the one in the photo above. **See margin.**

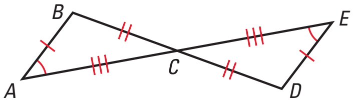
25. CAR STEREO A car stereo fits into a space in your dashboard. You want to buy a new car stereo, and it must fit in the existing space. What measurements need to be the same in order for the new stereo to be congruent to the old one? **length, width, and depth**

EXAMPLE 5 for Ex. 26

B **26. PROOF** Copy and complete the proof.

GIVEN ▶ $\overline{AB} \cong \overline{ED}$, $\overline{BC} \cong \overline{DC}$, $\overline{CA} \cong \overline{CE}$, $\angle BAC \cong \angle DEC$

PROVE ▶ $\triangle ABC \cong \triangle EDC$



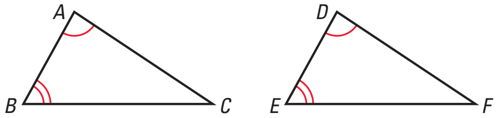
STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{ED}$, $\overline{BC} \cong \overline{DC}$, $\overline{CA} \cong \overline{CE}$, $\angle BAC \cong \angle DEC$	1. Given
2. $\angle BCA \cong \angle DCE$	2. _?_ **Vertical Angles Congruence Theorem**
3. _?_ **$\angle ABC \cong \angle EDC$**	3. Third Angles Theorem
4. $\triangle ABC \cong \triangle EDC$	4. _?_ **Definition of congruent figures**

27. ★ **SHORT RESPONSE** Suppose $\triangle ABC \cong \triangle DCB$, and the triangles share vertices at points B and C. Draw a figure that illustrates this situation. Is $\overline{AC} \parallel \overline{BD}$? *Explain.* **Yes; alternate interior angles are congruent; see margin for art.**

28. PROVING THEOREM 4.3 Use the plan to prove the Third Angles Theorem. **See margin.**

GIVEN ▶ $\angle A \cong \angle D$, $\angle B \cong \angle E$

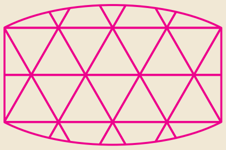
PROVE ▶ $\angle C \cong \angle F$



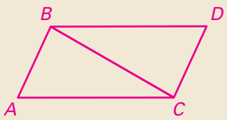
Plan for Proof Use the Triangle Sum Theorem to show that the sums of the angle measures are equal. Then use substitution to show $\angle C \cong \angle F$.

○ = See **WORKED-OUT SOLUTIONS** in Student Resources ★ = **STANDARDIZED TEST PRACTICE**

24. *Sample:*

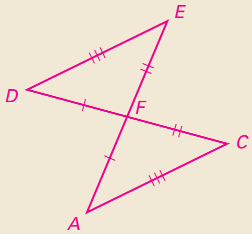


27.



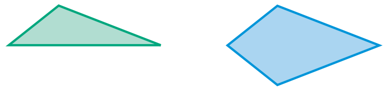
28. See Additional Answers.

29.



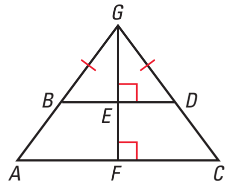
29. **REASONING** Given that $\triangle AFC \cong \triangle DFE$, must F be the midpoint of $\overline{AD}$ and $\overline{EC}$? Include a drawing with your answer. **No; see margin for art.**

30. ★ **SHORT RESPONSE** You have a set of tiles that come in two different shapes, as shown. You can put two of the triangular tiles together to make a quadrilateral that is the same size and shape as the quadrilateral tile.



Explain how you can find all of the angle measures of each tile by measuring only two angles. **Measure two angles of the triangle and use the Triangle Sum Theorem to find the third angle. The angles in the quadrilateral can be found using the angle measures of the triangle.**

31. **MULTI-STEP PROBLEM** In the diagram, quadrilateral $ABEF \cong$ quadrilateral $CDEF$.



a. *Explain* how you know that $\overline{BE} \cong \overline{DE}$ and $\angle ABE \cong \angle CDE$.

b. *Explain* how you know that $\angle GBE \cong \angle GDE$.

c. *Explain* how you know that $\angle GEB \cong \angle GED$. ***Sample answer:* All right angles are congruent.**

d. Do you have enough information to prove that $\triangle BEG \cong \triangle DEG$? *Explain*. **Yes; all corresponding parts of both triangles are congruent.**

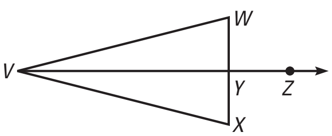
31a. Corresponding parts of congruent figures are congruent.

31b. They are supplementary to two congruent angles and therefore are congruent.

C **32.** **CHALLENGE** Use the diagram to write a proof.

GIVEN ▶ $\overline{WX} \perp \overrightarrow{VZ}$ at Y, Y is the midpoint of $\overline{WX}$, $\overline{VW} \cong \overline{VX}$, and $\overrightarrow{VZ}$ bisects $\angle WVX$.

PROVE ▶ $\triangle VWY \cong \triangle VXY$ **See margin.**

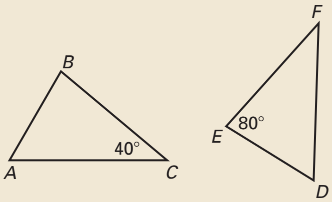


5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

In the diagram, $\triangle ABC \cong \triangle DEF$. Complete each statement.

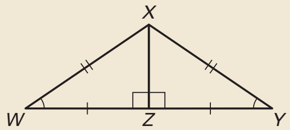


1. $m\angle A = \underline{?}$ **60°**

2. $\overline{FD} \cong \underline{?}$ $\overline{CA}$

3. $\triangle EDF \cong \underline{?}$ $\triangle BAC$

4. Write a congruence statement for the two small triangles. Explain your reasoning.



$\triangle WXZ \cong \triangle YXZ$; The diagram tells us that $\angle W \cong \angle Y$ and $\angle WZX \cong \angle YZX$. $\angle WXZ \cong \angle YXZ$ by the Third ∆s Thm. From the diagram $\overline{WX} \cong \overline{YX}$ and $\overline{WZ} \cong \overline{YZ}$, and $\overline{XZ} \cong \overline{XZ}$ by Refl. Prop. of $\cong$.

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book

32. See Additional Answers.

Alternative Strategy

In Example 4 in this lesson students proved overlapping triangles congruent. Overlapping triangles can be redrawn so they are next to each other. This makes it easier to see and mark the congruent sides and angles to determine why the triangles are congruent.

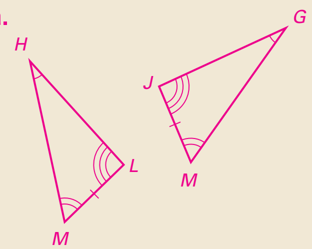
Avoiding Common Errors

In Practice 1, $\overline{JM}$ is marked congruent to $\overline{LM}$. Students may mark $\overline{HM}$ and $\overline{GM}$ congruent when they draw the triangles apart from each other. Remind them that it is just the small parts of these sides that are congruent.

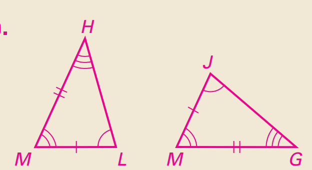
Study Strategy

Have students look back at the overlapping triangles in the original diagram for angles and sides that belong to both triangles. They can use the Reflexive Property of Congruence for either sides or angles.

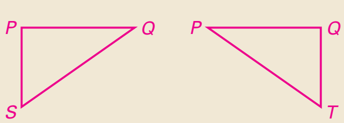
1a.



1b.



2.



PROBLEM SOLVING WORKSHOP LESSON 4.2

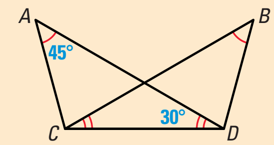
Using ALTERNATIVE METHODS

Another Way to Solve Example 4

MULTIPLE REPRESENTATIONS In Example 4, you used congruencies in triangles that overlapped. When you solve problems like this, it may be helpful to redraw the art so that the triangles do not overlap.

PROBLEM

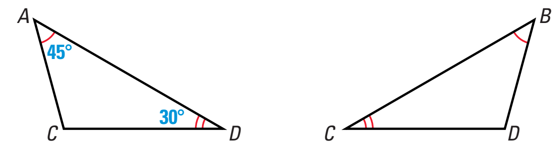
Find $m\angle BDC$.



METHOD

Drawing A Diagram

STEP 1 **Identify** the triangles that overlap. Then redraw them so that they are separate. Copy all labels and markings.



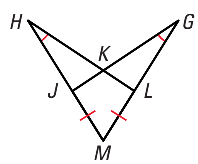
STEP 2 **Analyze** the situation. By the Triangle Sum Theorem, $m\angle ACD = 180° - 45° - 30° = 105°$.

Also, because $\angle A \cong \angle B$ and $\angle ADC \cong \angle BCD$, by the Third Angles Theorem, $\angle ACD \cong \angle BDC$, and $m\angle ACD = m\angle BDC = 105°$.

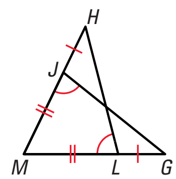
PRACTICE

1. **DRAWING FIGURES** Draw $\triangle HLM$ and $\triangle GJM$ so they do not overlap. Copy all labels and mark any known congruences. **a, b. See margin.**

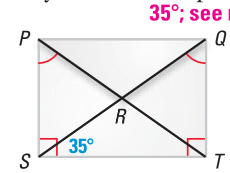
a.



b.



2. **ENVELOPE** Draw $\triangle PQS$ and $\triangle QPT$ so that they do not overlap. Find $m\angle PTS$. **35°; see margin for art.**



Rigid Motions in the Plane

MATERIALS • graph paper • ruler • protractor

QUESTION Which transformations are rigid motions?

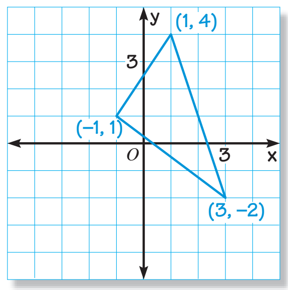
Transformations are functions that map points onto points. The result of transforming a figure is called its *image*. The original figure is called the *preimage*.

You can use function rules expressed with coordinate notation to describe some transformations.

EXPLORE 1 Use function rules for transformations

STEP 1 *Graph a triangle*

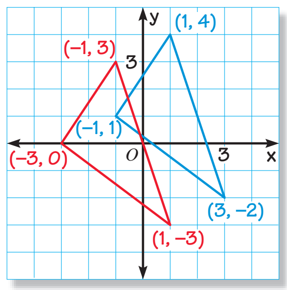
Graph the triangle whose vertices have the coordinates (1, 4), (3, −2), and (−1, 1).



STEP 2 *Transform the triangle*

Transform each vertex of the triangle using this function rule:

$(x, y) \rightarrow (x - 2, y - 1)$.



STEP 3 *Describe the transformation*

The transformation is a slide, or translation, 2 units left and 1 unit down. The image is congruent to the preimage.

STEP 4 *Repeat Steps 1–3 for different transformations*

For each function rule, draw a triangle and its image. Describe the transformation. Then tell whether the image is congruent to the preimage. **See margin.**

a. $(x, y) \rightarrow (-x, y)$

b. $(x, y) \rightarrow (2x, 2y)$

c. $(x, y) \rightarrow (-y, x)$

d. $(x, y) \rightarrow (x, 2y)$

1 PLAN AND PREPARE

Explore the Concept

- Students will investigate transformations in the coordinate plane, looking at lengths and angle measures.
- Students will decide whether a simple transformation is a rigid motion.
- This activity leads into further study of rigid motions and their relation to congruence.

Materials

Each student will need:

- graph paper
- ruler
- protractor

Recommended Time

Work activity: 15 min

Discuss results: 5 min

Grouping

Students should work individually.

2 TEACH

Tips for Success

In Step 4 of Explore 1, recommend that students use two colors when drawing their triangles on a coordinate grid. This will help them describe each transformation.

Key Question

- What must be true of a transformation for it to be a rigid motion? **The transformation must preserve length and angle measure.**

Explore 1, Step 4. Sample answers are given.

4a. The transformation is a flip, or reflection, in the *y*-axis. The image is congruent to the preimage.

4b. The transformation is an enlargement, or dilation. The image is not congruent to the preimage.

4c. The transformation is a 90° turn, or rotation, counterclockwise about the origin. The image is congruent to the preimage.

4d. The transformation is a stretch, or shear, in a vertical direction. The image is not congruent to the preimage.

Alternative Strategy

Explore 2 can be done on a coordinate plane if it is easier for students. Lengths can be confirmed using the distance formula if the endpoints have integer coordinates. However, only rotations of 90°, 180°, and 270° can be easily graphed. Reflections may need to be confined to reflections across an axis, across the line $y = x$, and across the line $y = -x$.

Key Discovery

Some transformations, but not all, preserve length and angle measure.

3 ASSESS AND RETEACH

Write a function rule using coordinate notation for the transformation. **Sample answers are given.**

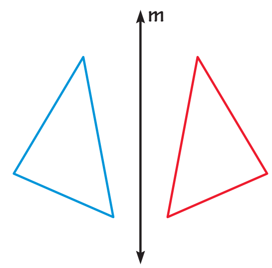
1. A transformation that preserves both length and angle measure **$(x, y) \rightarrow (x + 2, y - 3)$**
2. A transformation that preserves angle measure but *not* length **$(x, y) \rightarrow (2x, 2y)$**
3. A transformation that does *not* preserve length and does *not* preserve angle measure **$(x, y) \rightarrow (x + 2, 2y)$**

A *rigid motion* is a transformation that preserves length and angle measure.

EXPLORE 2 Determine if a transformation is a rigid motion

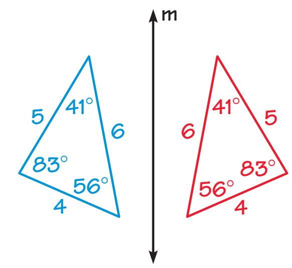
STEP 1 Draw transformation

Draw a triangle and its image after a reflection. You may want to fold your paper along the line of reflection to trace the triangle.



STEP 2 Measure sides and angles

Use a protractor to measure the angles and a ruler to measure the side lengths of the preimage and image.



STEP 3 Tell whether it is a rigid motion

The reflection preserves lengths and angle measures.
So, it is a rigid motion.

STEP 4 Repeat Steps 1–3 for different transformations

Draw a triangle and its image after an example of the transformation. Tell whether the transformation is a rigid motion.

a. translation (slide)**yes** **b.** rotation (turn) **yes** **c.** dilation (enlargement)**no**

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Use coordinate notation to write a function rule that describes reflection in the x-axis. Is this transformation a rigid motion? **$(x, y) \rightarrow (x, -y)$; yes**
2. Use coordinate notation to write a function rule that describes a rotation of 90° clockwise about the origin. Is this transformation a rigid motion? **$(x, y) \rightarrow (y, -x)$; yes**
3. Use coordinate notation to write a function rule that describes a rotation of 180° about the origin. Is this transformation a rigid motion? **$(x, y) \rightarrow (-x, -y)$; yes**
4. A triangle has vertices $(-1, 2)$, $(1, 3)$, and $(2, 0)$. What are the vertices of the image after a transformation described by the function rule $(x, y) \rightarrow (-2x, 3y)$? Is this transformation a rigid motion? *Explain.* **$(2, 6)$, $(-2, 9)$, $(-4, 0)$; no; neither lengths nor angles are preserved, so it is not a rigid motion.**
5. Which transformations in Explore 2 preserve length? **translation, reflection, rotation**
6. Which transformations in Explore 2 preserve angle measure? **translation, reflection, rotation, dilation**
7. Which transformations in Explore 2 are rigid motions? **translation, reflection, rotation**
8. If a transformation preserves area, is it always a rigid motion? If so, explain. If not, give a counterexample. **No; a transformation such as $(x, y) \rightarrow (2x, 0.5y)$ preserves area but does not preserve length or angle measure, so it is not a rigid motion.**

4.3 Relate Transformations and Congruence

Before You identified congruent figures.

Now You will use transformations to show congruence.

Why So you can complete an architect's drawing, as in Ex. 28.

Key Vocabulary
• rigid motion

Transformations in the plane move or change a figure to produce a new figure. A **rigid motion** is a transformation that preserves length, angle measure, and area. A rigid motion is also called an *isometry. Translations, reflections,* and *rotations* are examples of rigid motions.

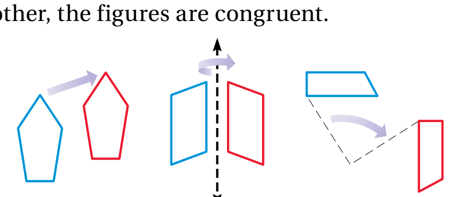
Recall that two figures are *congruent* if and only if the corresponding sides and the corresponding angles are congruent. Two geometric figures are congruent if and only if there is a rigid motion or a combination of rigid motions that move one of the figures onto the other.

READ VOCABULARY
Translations are also known as *slides,* reflections are also known as *flips,* and rotations are also known as *turns.*

KEY CONCEPT *For Your Notebook*

Congruent Figures and Transformations

Two figures are congruent if and only if one or more rigid motions can be used to move one figure onto the other. If any combination of translations, reflections, and rotations can be used to move one shape onto the other, the figures are congruent.

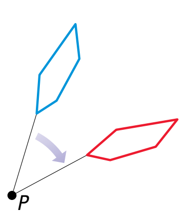


translation reflection rotation

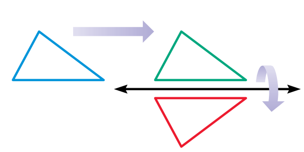
EXAMPLE 1 **Describe rigid motions to show congruence**

Describe the transformation(s) you can use to move the blue figure onto the red figure.

a.



b.



Solution

a. rotation about *P*

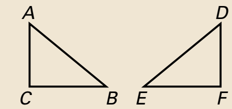
b. translation and then reflection

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

1. In the diagram, $\triangle ABC \cong \triangle DEF$. Name the pairs of corresponding angles and corresponding sides.



$\angle A$ and $\angle D$, $\angle B$ and $\angle E$, $\angle C$ and $\angle F$, $\overline{AB}$ and $\overline{DE}$, $\overline{BC}$ and $\overline{EF}$, $\overline{CA}$ and $\overline{FD}$

Find the image of the points after the transformation whose rule is given.

2. $(-4, 3)$, $(1, 4)$, $(2, -5)$,
$(x, y) \rightarrow (x - 2, y + 1)$
$(-6, 4)$, $(-1, 5)$, $(0, -4)$

3. $(-2, 1)$, $(3, 5)$, $(-1, -4)$,
$(x, y) \rightarrow (-y, x)$
$(-1, -2)$, $(-5, 3)$, $(4, -1)$

4. $(2, -4)$, $(-3, 4)$, $(4, -2)$,
$(x, y) \rightarrow (-x, y)$
$(-2, -4)$, $(3, 4)$, $(-4, -2)$

Pacing

Basic: 1 day
Average: 1 day
Advanced: 1 day
Block: 0.5 block

2 FOCUS AND MOTIVATE

Essential Question

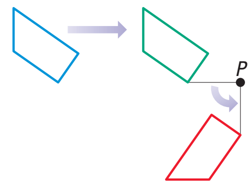
Big Idea 2

How do you identify a rigid motion in the plane? **Tell students they will learn how to answer this question by using transformations in the plane that preserve length and angle measure.**

✓ GUIDED PRACTICE for Example 1

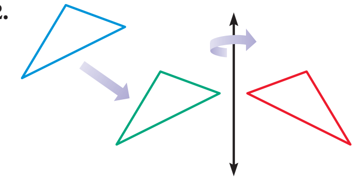
Describe the transformation(s) you can use to move the blue figure onto the red figure.

1.



translation and then rotation

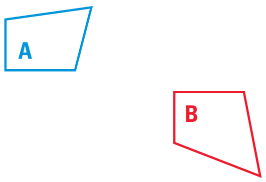
2.



translation and then reflection

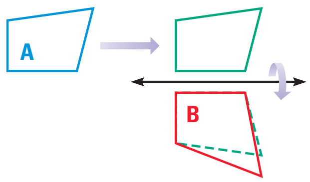
EXAMPLE 2 Show figures are not congruent

Explain why figure A and figure B are not congruent using transformations.



Solution

Translate and then reflect figure A to compare it to figure B. Although some of the corresponding sides are congruent, the corresponding angles are not all congruent. So, the figures are not congruent.



EXAMPLE 3 Move one figure in a pattern onto another

QUILTING The quilt shown is made using a repeating pattern.

a. Describe a transformation that moves figure A onto figure B.

b. Explain why figures C and D are congruent by using the marked angles and sides.

Solution

a. A translation or a rotation will move figure A onto figure B.

b. Figures C and D are congruent because all pairs of corresponding sides and all pairs of corresponding angles are congruent.

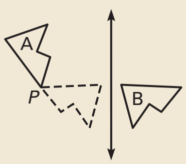
Motivating the Lesson

Ask students to draw examples of flips, turns, and slides as they remember them from previous instruction. Have them identify some real-life examples of flips, turns, and slides.

3 TEACH

Extra Example 1

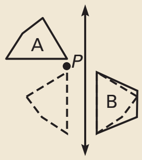
Describe the transformation(s) you can use to move figure A onto figure B.



rotation about *P* and then reflection

Extra Example 2

Explain why figure A and figure B are not congruent using transformations.



Rotate figure A about point *P* and then reflect that image to compare it to figure B. Although one pair of corresponding sides is congruent, none of the corresponding angles are congruent. So, the figures are not congruent.

Differentiated Instruction

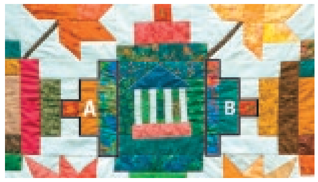
English Language Learners *Rigid motion* and *isometry* are terms that can be used interchangeably. Ask students to keep a journal listing of all the terms related to transformations, including diagrams of each term.

See also the *Differentiated Instruction Resources* for more strategies.

✓ GUIDED PRACTICE for Examples 2 and 3

Tell whether the two figures in the quilt are *congruent* or *not congruent*. If they are congruent, describe the transformation(s) you can use to move figure A onto figure B.

3.



not congruent

4.

congruent; reflection

4.3 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** for Exs. 11, 17, and 21

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6–9, 21, 25, 28

A

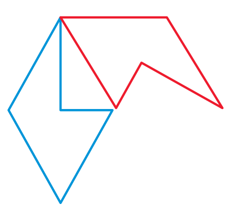
1. VOCABULARY Examples of transformations that are rigid motions are __?__, __?__, and __?__. translations, reflections, rotations

2. ★ WRITING *Explain* why a transformation that maps one figure onto a congruent figure is a rigid motion. A transformation that maps one figure onto a congruent figure preserves lengths and angle measures, so it is a rigid motion.

EXAMPLE 1 for Exs. 3–8

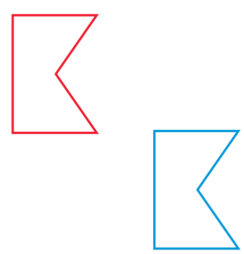
IDENTIFYING TRANSFORMATIONS Identify the transformation you can use to move the blue figure onto the red figure.

3.



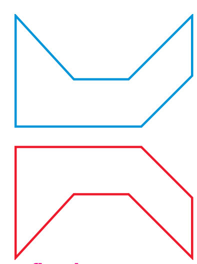
rotation

4.



translation

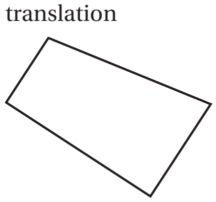
5.



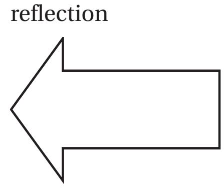
reflection

★ OPEN-ENDED MATH Copy the figure. Draw an example of the effect of the given transformation on the figure. 6–8. Check students' drawings.

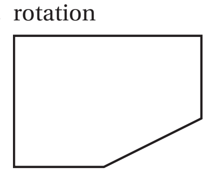
6. translation



7. reflection

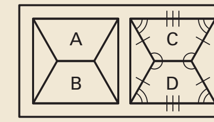


8. rotation



Extra Example 3

The quilt shown is made using a repeating pattern.



a. Describe a transformation that moves figure A onto figure B. **A reflection or rotation will move figure A onto figure B.**

b. Explain why figures C and D are congruent by using the marked angles and sides. **Figures C and D are congruent because all pairs of corresponding sides and all pairs of corresponding angles are congruent.**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you identify a rigid motion in the plane?

- **Rigid motions preserve length and angle measure.**
- **Two figures are congruent if and only if one or more rigid motions can be used to move one figure onto the other.**
- **Translations, reflections, and rotations are rigid motions.**

If the transformation of a figure results in an image that has corresponding angles and corresponding sides congruent to those in the preimage, then the transformation is a rigid motion.

Differentiated Instruction

Kinesthetic Learners You might wish to help your kinesthetic learners understand congruence in terms of rigid motions by providing them with large cardboard triangles they can use to perform translations, reflections, and rotations.

See also the *Differentiated Instruction Resources* for more strategies.

4 PRACTICE AND APPLY

Assignment Guide

Basic:
Day 1:
Exs. 1–9, 11–17 odd, 20–22

Average:
Day 1:
Exs. 1–9, 11, 13, 15–17, 20–23, 26–28

Advanced:
Day 1:
Exs. 3–9, 10–16 even, 17–19, 21, 23–26, 28–29

Block:
Exs. 1–9, 11, 13, 15–17, 20–23, 26–28

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 6, 9, 13, 20
Average: 4, 7, 11, 21, 23
Advanced: 5, 8, 16, 24, 26

Extra Practice

• Practice B in Chapter Resources

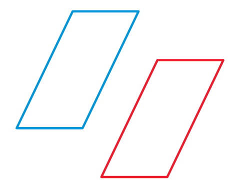
Practice Worksheet

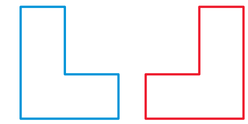
An easily-readable reduced practice page can be found at the beginning of this chapter.

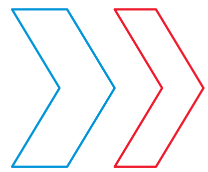
10. No; a reflection maps one side to a congruent side, but other sides are not congruent.
11. yes; reflection in the line $y = x$
12. yes; translation 3 units right and 2 units down

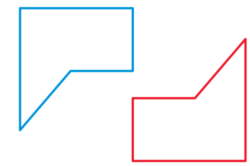
EXAMPLES 2 AND 3 for Exs. 9–17 **B**

9. ★ **MULTIPLE CHOICE** Which is *not* an example of a rigid motion? **C**

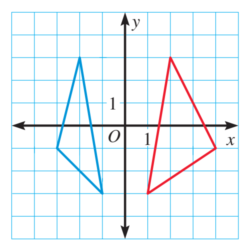
(A) 

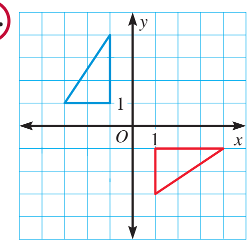
(B) 

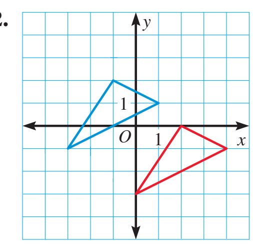
(C) 

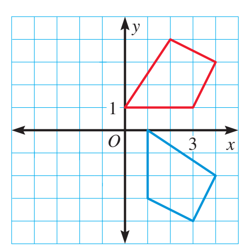
(D) 

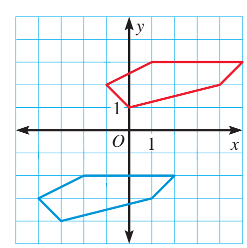
SHOWING FIGURES CONGRUENT **Tell whether a rigid motion can move the blue figure onto the red figure. If so, describe the transformation(s) that you can use. If not, explain why the figures are not congruent.** 10–15. See margin.

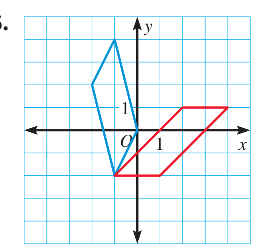
10. 

11. 

12. 

13. 

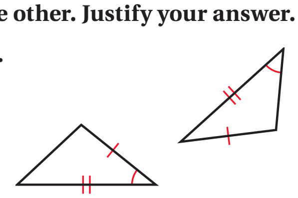
14. 

15. 

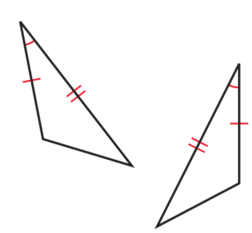
16. Jerome describes a transformation in the coordinate plane using the notation $(x, y) \to (x + 3, y - 1)$. *Explain* why this is a rigid motion. See margin.

17. Jen describes a transformation in the coordinate plane using the notation $(x, y) \to (x - 1, 2y)$. *Explain* why this is *not* a rigid motion. See margin.

C **CHALLENGE** **Determine whether a rigid motion can move one triangle onto the other. Justify your answer.**

18. 

No; check students' drawings.

19. 

Yes; check students' drawings; a translation followed by a reflection, or a rotation followed by a reflection

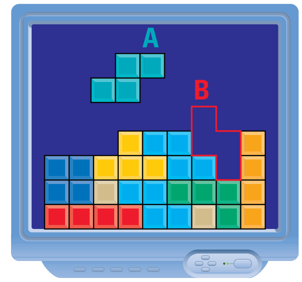
13. yes; rotation 90° counterclockwise about the origin
14. yes; translation 3 units right and 5 units up
15. No; a rotation does not map one figure onto the other, because corresponding sides lengths are not congruent.
16. *Sample answer:* The function rule describes a translation 3 units to the right and 1 unit down. A translation is a rigid motion.
17. *Sample answer:* The function rule moves points 1 unit to the left and then stretches points vertically away from the x-axis. The transformation is not a rigid motion, because lengths and angles are not preserved. The triangle with vertices (0, 0), (1, 0), and (1, 1) is transformed to a taller triangle with vertices (–1, 0), (0, 0), and (0, 2).

PROBLEM SOLVING

EXAMPLE 3 A
for Exs. 20–26

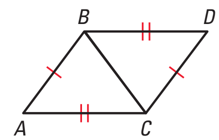
20. GAME SOFTWARE In a game, the goal is to move shapes into congruent spaces where they will fit so that completed rows can be eliminated. Describe a combination of transformations that can be used to move game piece A into congruent space B.

90° rotation (either way), followed by translation across and down



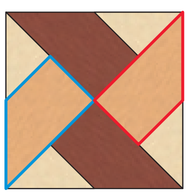
21. ★ **SHORT RESPONSE** Describe a way in which $\triangle ABC$ can be moved onto $\triangle DCB$ using just one transformation. Then describe a way in which $\triangle ABC$ can be moved onto $\triangle DCB$ using exactly two transformations.

180° rotation around the midpoint of $\overline{BC}$; reflection across $\overline{BC}$ followed by reflection across the perpendicular bisector of $\overline{BC}$



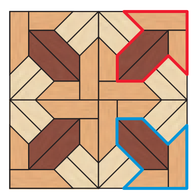
FLOORING Designs for floor tiles are shown below. Describe a rigid motion or combination of rigid motions that can be used to move the blue figure onto the red figure.

22.



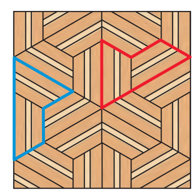
180° rotation

23.



90° rotation counterclockwise

24.



120° rotation clockwise, followed by translation

B

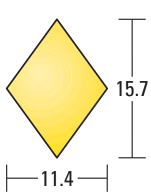
25. ★ **OPEN-ENDED MATH** Create a design using a combination of translations, reflections, or rotations. **Check students' designs.**

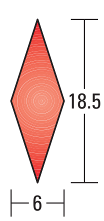
26. PENROSE TILES The mathematician Roger Penrose investigated the patterns that can be made with tiles like the ones shown below.

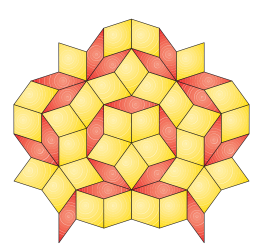
a. *Choose* two tiles of the same color in the pattern and show how to map one tile onto the other using a rotation.

b. *Describe* the rotation angle and center.

c. *Explain* how you calculated the rotation angle.







a–c. See margin.

Teaching Strategy

Exercises 10–15 Some students may have difficulty recognizing a rigid motion in the plane using coordinates. Suggest they use the distance formula to check that the corresponding sides of the preimage and image are congruent.

Study Strategy

Exercises 16–17 You may wish to have students provide examples of transformations in the plane in their explanations. Suggest that they save their examples for study-ing purposes.

Reading Strategy

Exercises 22–24 Ask students to focus on the color of the figures they are describing. The blue figure is the preimage figure throughout these exercises. The red figure is the resulting image figure.

26a. Check students' rotations.

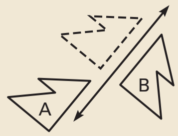
26b. *Sample answer:* Yellow tiles that share an edge can be rotated either 72° around the vertex of the smaller angle or 108° around the vertex of the larger angle. Red tiles that share an edge can be rotated 36° around the vertex of the smaller angle or 144° around the vertex of the larger angle.

26c. *Sample answer:* You can calculate the angles of the tiles by observing how many of each type meet at various vertices in the design.

5 ASSESS AND RETEACH

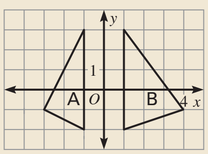
Daily Homework Quiz

1. Identify the transformation(s) you can use to move figure A onto figure B.



translation, then reflection

2. Tell whether a rigid motion can move figure A onto figure B. Explain.



No; a reflection maps one side to a congruent side, but the other sides are not congruent.

Diagnosis/Remediation

- Practice B in Chapter Resources
- Study Guide in Chapter Resources

Challenge

Additional challenge is available in the Chapter Resources.

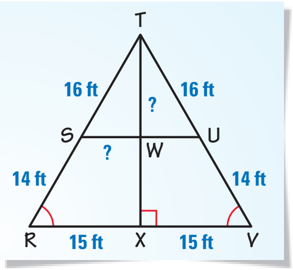
28a. The rigid motion of reflection across a vertical line maps $\triangle RTX$ onto $\triangle VTX$, so $\triangle RTX \cong \triangle VTX$.

28b. The same rigid motion that maps $\triangle RTX$ onto $\triangle VTX$ also maps $\triangle STW$ onto $\triangle UTW$, so $\triangle STW \cong \triangle UTW$. Because the triangles are congruent, the corresponding sides are congruent, so $SW = UW$.

28c. $SW = 8$ ft; by the Pythagorean Theorem, $TW \approx 13.9$ ft

27. **CLOTHING DESIGN** A clothing manufacturer needs two panels cut from cloth that are reflections of each other to create part of a dress. *Explain* why folding the fabric in half and cutting both pieces together will produce the two panels.
Cutting both pieces together will make all corresponding sides and angles of the pattern congruent.

C 28. ★ **EXTENDED RESPONSE** The diagram shows an architect's preliminary design for an A-frame house. **See margin.**



a. **Interpret** Use rigid motions to explain how the architect knows $\triangle RTX \cong \triangle VTX$.

b. **Reason** Explain how the architect can use rigid motions to conclude that $SW = UW$.

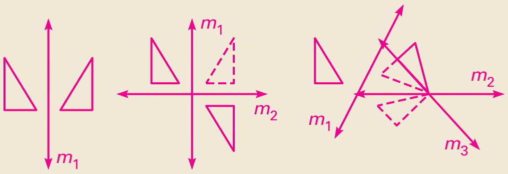
c. **Calculate** The architect knows that the proportion $\frac{TS}{SW} = \frac{TR}{RX}$ is true. Find the lengths SW and TW.

29. **CHALLENGE** Draw examples of two congruent triangles that can be mapped onto each other by the given transformation(s). Show the lines(s) of reflection you use. **See margin.**

a. exactly one reflection b. exactly two reflections c. exactly three reflections

See **EXTRA PRACTICE** in Student Resources

29a–c. *Sample answer.*



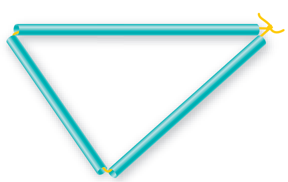
Investigate Congruent Figures

MATERIALS • straws • string • ruler • protractor

QUESTION How much information is needed to tell whether two figures are congruent?

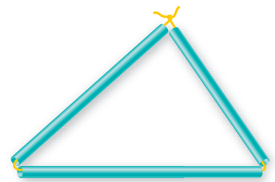
EXPLORE 1 Compare triangles with congruent sides

STEP 1



Make a triangle Cut straws to make side lengths of 8 cm, 10 cm, and 12 cm. Thread the string through the straws. Make a triangle by connecting the ends of the string.

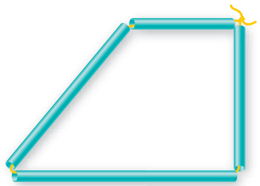
STEP 2



Make another triangle Use the same length straws to make another triangle. If possible, make it different from the first. Compare the triangles. What do you notice?

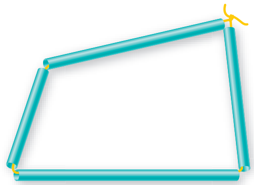
EXPLORE 2 Compare quadrilaterals with congruent sides

STEP 1



Make a quadrilateral Cut straws to make side lengths of 5 cm, 7 cm, 9 cm, and 11 cm. Thread the string through the straws. Make a quadrilateral by connecting the string.

STEP 2



Make another quadrilateral Make a second quadrilateral using the same length straws. If possible, make it different from the first. Compare the quadrilaterals. What do you notice?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Can you make two triangles with the same side lengths that are different shapes? *Justify* your answer. **No. *Sample answer:* In the activity once the triangle lengths were established it was impossible to create two different triangles.**
2. If you know that three sides of a triangle are congruent to three sides of another triangle, can you say the triangles are congruent? *Explain.* **Yes. *Sample answer:* The activity indicates that this is true.**
3. Can you make two quadrilaterals with the same side lengths that are different shapes? *Justify* your answer. **Yes. *Sample answer:* In the activity it was possible to change the angles in the quadrilateral thus allowing for more than one figure.**
4. If four sides of a quadrilateral are congruent to four sides of another quadrilateral, can you say the quadrilaterals are congruent? *Explain.* **No. *Sample answer:* The activity established that this would not be sufficient to prove congruence.**

1 PLAN AND PREPARE

Explore the Concept

- Students will investigate how many congruent sides are needed for two triangles to be congruent.
- This activity leads into proving triangles congruent by SSS in this lesson.

Materials

Each student or group of students will need:

- straws
- string
- ruler, protractor, scissors

Recommended Time

Work activity: 15 min

Discuss results: 5 min

Grouping

Students can work individually or in groups of two. If students work in groups, one can cut the straws and the other can thread the string.

2 TEACH

Tips for Success

Use straws that are big enough so that students do not have trouble threading the string through them.

Key Question

- Are triangles congruent if one can be turned or flipped over to match the other? **yes**

Alternative Strategy

Demonstrate with straws on the overhead projector.

Key Discovery

Three pairs of congruent sides make triangles congruent.

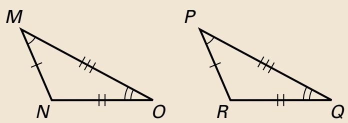
3 ASSESS AND RETEACH

1. If you have only information about sides, how much information is necessary to prove two triangles congruent? **three pairs of congruent sides**

1 PLAN AND PREPARE

Warm-Up Exercises

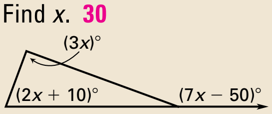
Also available online



1. Write a congruence statement. **$\triangle MNO \cong \triangle PRQ$**

2. How do you know that $\angle N \cong \angle R$? **Third $\triangle$s Thm.**

3. Find x. **30**



Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 1 day

Average: 1 day

Advanced: 1 day

Block: 0.5 block with previous lesson

• See *Teaching Guide/Lesson Plan.*

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 2

How can you use side lengths to prove triangles congruent? **Tell students they will see how to answer this question by learning the SSS Congruence Postulate.**

4.4 Prove Triangles Congruent by SSS

Before You used the definition of congruent figures.

Now You will use the side lengths to prove triangles are congruent.

Why So you can determine if triangles in a tile floor are congruent, as in Ex. 22.

Key Vocabulary
- **congruent figures**
- **corresponding parts**

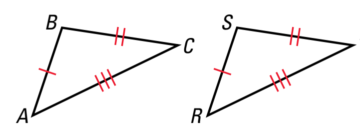
In the Activity *Investigate Congruent Figures*, you saw that there is only one way to form a triangle given three side lengths. In general, any two triangles with the same three side lengths must be congruent.

POSTULATE *For Your Notebook*

POSTULATE 19 Side-Side-Side (SSS) Congruence Postulate

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If **S**ide $\overline{AB} \cong \overline{RS}$,
Side $\overline{BC} \cong \overline{ST}$, and
Side $\overline{CA} \cong \overline{TR}$,
then $\triangle ABC \cong \triangle RST$.

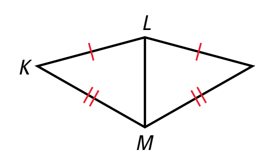


EXAMPLE 1 Use the SSS Congruence Postulate

Write a proof.

GIVEN ▶ $\overline{KL} \cong \overline{NL}$, $\overline{KM} \cong \overline{NM}$

PROVE ▶ $\triangle KLM \cong \triangle NLM$



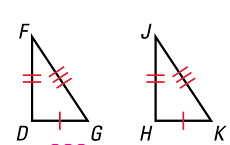
Proof It is given that $\overline{KL} \cong \overline{NL}$ and $\overline{KM} \cong \overline{NM}$. By the Reflexive Property, $\overline{LM} \cong \overline{LM}$. So, by the SSS Congruence Postulate, $\triangle KLM \cong \triangle NLM$.

Animated Geometry at my.hrw.com

✓ GUIDED PRACTICE for Example 1

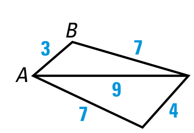
Decide whether the congruence statement is true. *Explain* your reasoning.

1. $\triangle DFG \cong \triangle HJK$



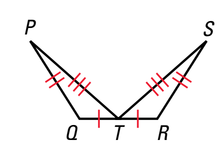
yes; SSS

2. $\triangle ACB \cong \triangle CAD$



2. No; corresponding sides $\overline{AB}$ and $\overline{CD}$ are not congruent.

3. $\triangle QPT \cong \triangle RST$

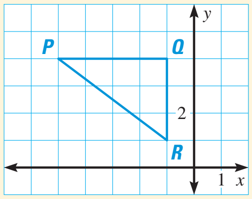


yes; SSS

Mark Downey/Getty Images

★ EXAMPLE 2 Standardized Test Practice

Which are the coordinates of the vertices of a triangle congruent to $\triangle PQR$?



Ⓐ $(-1, 1), (-1, 5), (-4, 5)$

Ⓑ $(-2, 4), (-7, 4), (-4, 6)$

Ⓒ $(-3, 2), (-1, 3), (-3, 1)$

Ⓓ $(-7, 7), (-7, 9), (-3, 7)$

Solution

By counting, $PQ = 4$ and $QR = 3$. Use the Distance Formula to find PR.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PR = \sqrt{(-1 - (-5))^2 + (1 - 4)^2} = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$$

By the SSS Congruence Postulate, any triangle with side lengths 3, 4, and 5 will be congruent to $\triangle PQR$. The distance from $(-1, 1)$ to $(-1, 5)$ is 4. The distance from $(-1, 5)$ to $(-4, 5)$ is 3. The distance from $(-1, 1)$ to $(-4, 5)$ is $\sqrt{(5 - 1)^2 + ((-4) - (-1))^2} = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$.

▶ The correct answer is A. Ⓐ Ⓑ Ⓒ Ⓓ

ELIMINATE CHOICES

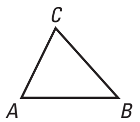
Once you know the side lengths of $\triangle PQR$, look for pairs of coordinates with the same x-coordinates or the same y-coordinates. In Choice C, $(-3, 2)$ and $(-3, 1)$ are only 1 unit apart. You can eliminate D in the same way.

✓ GUIDED PRACTICE for Example 2

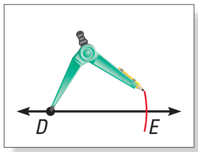
4. $\triangle JKL$ has vertices $J(-3, -2)$, $K(0, -2)$, and $L(-3, -8)$. $\triangle RST$ has vertices $R(10, 0)$, $S(10, -3)$, and $T(4, 0)$. Graph the triangles in the same coordinate plane and show that they are congruent. **$KJ = SR = 3$, $JL = RT = 6$, $LK = TS = 3\sqrt{5}$; see margin for art.**

ACTIVITY COPY A TRIANGLE

Follow the steps below to construct a triangle that is congruent to $\triangle ABC$.

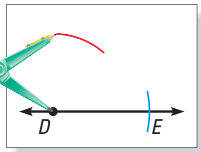


STEP 1



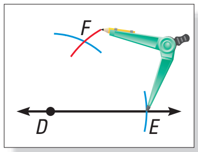
Construct $\overline{DE}$ so that it is congruent to $\overline{AB}$.

STEP 2



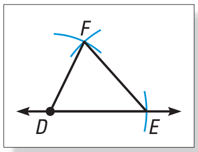
Open your compass to the length AC. Use this length to draw an arc with the compass point at D.

STEP 3



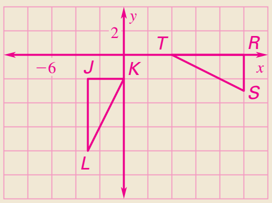
Draw an arc with radius BC and center E that intersects the arc from Step 2. Label the intersection point F.

STEP 4



Draw $\triangle DEF$. By the SSS Congruence Postulate, $\triangle ABC \cong \triangle DEF$.

4.



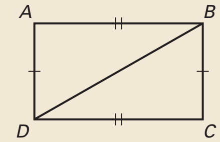
Motivating the Lesson

Tell students that a builder needs to order a triangular slab of marble for a section of museum wall. The builder knows the side lengths for the slab but does not have the angle measures. Tell students that in this lesson, they will learn whether the builder has enough information to order the slab she needs.

3 TEACH

Extra Example 1

Prove $\triangle ABD \cong \triangle CDB$.



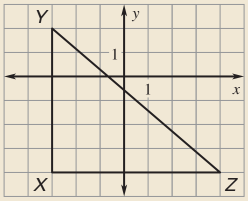
It is given that $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$. By the Refl. Prop. of $\cong$ Segments, $\overline{BD} \cong \overline{BD}$. So, by the SSS $\cong$ Post., $\triangle ABD \cong \triangle CDB$.

Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for **Example 1**. This activity is also part of **Power Presentations**.

Extra Example 2

Which are the coordinates of the vertices of a triangle congruent to $\triangle XYZ$? **B**



Ⓐ $(6, 2), (0, -6), (6, -5)$

Ⓑ $(5, 1), (-1, -6), (5, -6)$

Ⓒ $(4, 0), (-1, -7), (4, -7)$

Ⓓ $(3, -1), (-3, -7), (3, -8)$

EXAMPLE 3 Solve a real-world problem

STRUCTURAL SUPPORT Explain why the bench with the diagonal support is stable, while the one without the support can collapse.

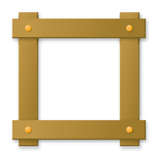
Solution

The bench with a diagonal support forms triangles with fixed side lengths. By the SSS Congruence Postulate, these triangles cannot change shape, so the bench is stable. The bench without a diagonal support is not stable because there are many possible quadrilaterals with the given side lengths.

✓ GUIDED PRACTICE for Example 3

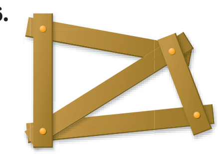
Determine whether the figure is stable. *Explain* your reasoning.

5.



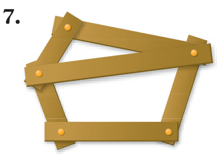
Not stable; a figure without diagonal support is not stable.

6.



Stable; the figure has diagonal support with fixed side lengths.

7.



Not stable; the lower half of the figure does not have diagonal support.

4.4 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 7, 9, and 25

★ = **STANDARDIZED TEST PRACTICE** Exs. 16, 17, and 28

SKILL PRACTICE

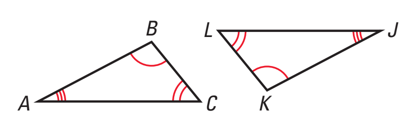
A **VOCABULARY** Tell whether the angles or sides are *corresponding angles*, *corresponding sides*, or *neither*.

1. $\angle C$ and $\angle L$
corresponding angles

2. $\overline{AC}$ and $\overline{JK}$ neither

3. $\overline{BC}$ and $\overline{KL}$
corresponding sides

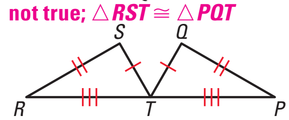
4. $\angle B$ and $\angle L$
neither



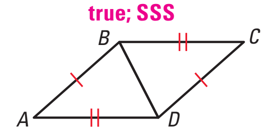
EXAMPLE 1
for Exs. 5–7

DETERMINING CONGRUENCE Decide whether the congruence statement is true. *Explain* your reasoning.

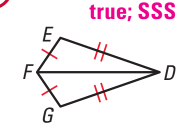
5. $\triangle RST \cong \triangle TQP$
not true; $\triangle RST \cong \triangle PQT$



6. $\triangle ABD \cong \triangle CDB$
true; SSS



7. $\triangle DEF \cong \triangle DGF$
true; SSS

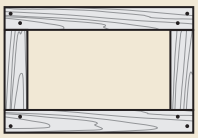


Activity Note

This activity reinforces the idea that the size and shape of a triangle are completely determined by the side lengths.

Extra Example 3

The opposite sides of the gate are the same length. How would you put a brace on the gate to be sure that the gate keeps its shape? Explain.



Diagonally, since the triangles will be rigid and, by the SSS ≅ Post., cannot change shape.

Closing the Lesson

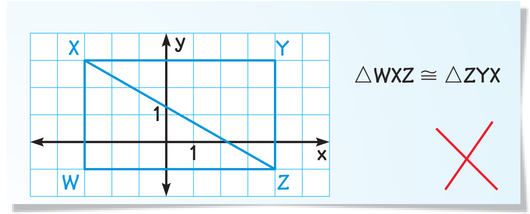
Have students summarize the major points of the lesson and answer the Essential Question: How can you use side lengths to prove triangles congruent?

- **Two triangles with the same side lengths are congruent by the SSS Congruence Postulate.**

Show that the sides can be matched so that all three pairs of corresponding sides are congruent.

EXAMPLE 2
for Exs. 8–12

8. ERROR ANALYSIS *Describe* and correct the error in writing a congruence statement for the triangles in the coordinate plane. **The triangle vertices do not correspond. *Sample answer:* $\triangle WXZ \cong \triangle YZX$.**



ALGEBRA **Use the given coordinates to determine if $\triangle ABC \cong \triangle DEF$.**

9. $A(-2, -2)$, $B(4, -2)$, $C(4, 6)$, $D(5, 7)$, $E(5, 1)$, $F(13, 1)$ **congruent**

10. $A(-2, 1)$, $B(3, -3)$, $C(7, 5)$, $D(3, 6)$, $E(8, 2)$, $F(10, 11)$ **not congruent**

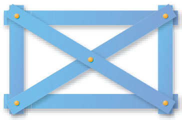
11. $A(0, 0)$, $B(6, 5)$, $C(9, 0)$, $D(0, -1)$, $E(6, -6)$, $F(9, -1)$ **congruent**

12. $A(-5, 7)$, $B(-5, 2)$, $C(0, 2)$, $D(0, 6)$, $E(0, 1)$, $F(4, 1)$ **not congruent**

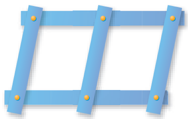
EXAMPLE 3
for Exs. 13–15

USING DIAGRAMS **Decide whether the figure is stable. *Explain*.**

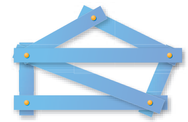
13.



14.



15.



13. Stable; the figure has diagonal support with fixed side lengths. B

14. Not stable; a figure without diagonal support is not stable.

15. Stable; the figure has diagonal support with fixed side lengths.

16. ★ **MULTIPLE CHOICE** Let $\triangle FGH$ be an equilateral triangle with point J as the midpoint of $\overline{FG}$. Which of the statements below is *not* true? **B**

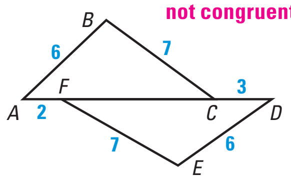
Ⓐ $\overline{FH} \cong \overline{GH}$ Ⓑ $\overline{FJ} \cong \overline{FH}$ Ⓒ $\overline{FJ} \cong \overline{GJ}$ Ⓓ $\triangle FHJ \cong \triangle GHJ$

17. ★ **MULTIPLE CHOICE** Let $ABCD$ be a rectangle separated into two triangles by $\overline{DB}$. Which of the statements below is *not* true? **B**

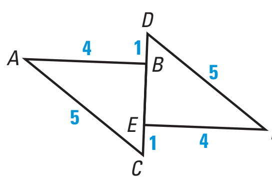
Ⓐ $\overline{AD} \cong \overline{CB}$ Ⓑ $\overline{AB} \cong \overline{AD}$ Ⓒ $\overline{AB} \cong \overline{CD}$ Ⓓ $\triangle DAB \cong \triangle BCD$

APPLYING SEGMENT ADDITION **Determine whether $\triangle ABC \cong \triangle DEF$. If they are congruent, write a congruence statement. *Explain* your reasoning.**

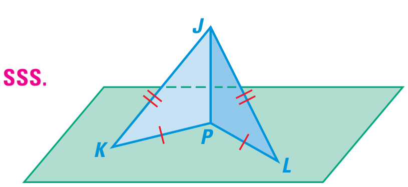
18. **not congruent; $CA \neq FD$**



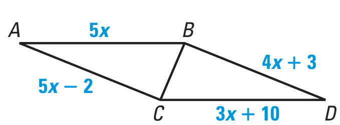
19. **Not congruent; the congruence statement should read $\triangle ABC \cong \triangle FED$.**



20. 3-D FIGURES In the diagram, $\overline{PK} \cong \overline{PL}$ and $\overline{JK} \cong \overline{JL}$. Show that $\triangle JPK \cong \triangle JPL$.
Since $\overline{JP} \cong \overline{JP}$ the triangles are congruent by SSS.



21. CHALLENGE Find all values of x that make the triangles congruent. *Explain*.



21. 5; setting $5x = 4x + 3$ and $5x - 2 = 3x + 10$ yields $x = 3$ and $x = 6$ which is inconsistent. Setting $5x = 3x + 10$ and $5x - 2 = 4x + 3$ yields $x = 5$ in both equations and is the answer. C

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–17, 22–27

Average:
Day 1:
Exs. 1–8, 10–14 even, 16–20, 22–29

Advanced:
Day 1:
Exs. 1–4, 7, 8, 11, 12, 14–30*

Block:
Exs. 1–8, 10–14 even, 16–20, 22–29 (with previous lesson)

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 5, 10, 13, 22, 23
Average: 6, 10, 14, 22, 23
Advanced: 7, 12, 15, 22, 23

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

Study Strategy

Exercises 25, 27 These exercises involve proving overlapping triangles congruent. Draw and label the triangles apart from each other. It may be easier to see the corresponding sides.

Mathematical Reasoning

Exercise 26 Students should ask themselves what conclusion follows from the given information that E is the midpoint of $\overline{BD}$. They should mark the diagram accordingly and be sure to include the statement $\overline{BE} \cong \overline{DE}$ as one step in the proof.

Internet Reference

Exercise 29 Addition information about the construction and layout of a baseball field can be found at edis.ifas.ufl.edu/EP092

24. Statements (Reasons)

1. $\overline{GH} \cong \overline{JK}$, $\overline{HJ} \cong \overline{KG}$ (Given)
2. $\overline{JG} \cong \overline{GJ}$ (Reflexive Property of Congruence)
3. $\triangle GHJ \cong \triangle JKG$ (SSS)

25. Statements (Reasons)

1. $\overline{WX} \cong \overline{VZ}$, $\overline{WY} \cong \overline{VY}$, $\overline{YZ} \cong \overline{YX}$ (Given)
2. $\overline{WV} \cong \overline{VW}$ (Reflexive Property of Congruence)
3. $WY = VY$, $YZ = YX$ (Definition of segment congruence)
4. $WY + YZ = VY + YZ$ (Addition Property of Equality)
5. $WY + YZ = VY + YX$ (Substitution Property of Equality)
6. $WZ = VX$ (Segment Addition Postulate)
7. $\overline{WZ} \cong \overline{VX}$ (Definition of segment congruence)
8. $\triangle VWX \cong \triangle WVZ$ (SSS)

26. Statements (Reasons)

1. $\overline{AE} \cong \overline{CE}$, $\overline{AB} \cong \overline{CD}$, E is the midpoint of $\overline{BD}$. (Given)
2. $\overline{BE} \cong \overline{DE}$ (Definition of midpoint)
3. $\triangle EAB \cong \triangle ECD$ (SSS)

PROBLEM SOLVING

EXAMPLE 1 A for Ex. 22

22. TILE FLOORS You notice two triangles in the tile floor of a hotel lobby. You want to determine if the triangles are congruent, but you only have a piece of string. Can you determine if the triangles are congruent? *Explain.* **Yes; use the string to measure each side of one triangle and then measure the sides of the second triangle to see if they are congruent to the corresponding sides of the first triangle.**

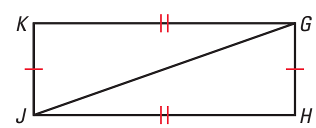
EXAMPLE 3 for Ex. 23

23. GATES Which gate is stable? *Explain* your reasoning. **See margin.**

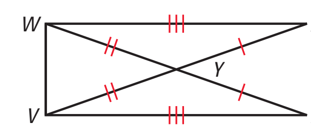
23. Gate 1. *Sample Answer:* Gate 1 has a diagonal support that forms two triangles with fixed side lengths, and these triangles cannot change shape. Gate 2 is not stable because the gate is a quadrilateral which can take many different shapes.

PROOF Write a proof. **24–27. See margin.**

24. **GIVEN** ▶ $\overline{GH} \cong \overline{JK}$, $\overline{HJ} \cong \overline{KG}$
PROVE ▶ $\triangle GHJ \cong \triangle JKG$

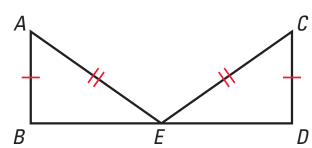


25. **GIVEN** ▶ $\overline{WX} \cong \overline{VZ}$, $\overline{WY} \cong \overline{VY}$, $\overline{YZ} \cong \overline{YX}$
PROVE ▶ $\triangle VWX \cong \triangle WVZ$

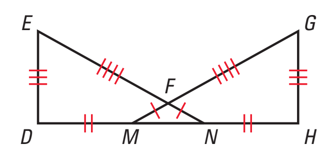


B

26. **GIVEN** ▶ $\overline{AE} \cong \overline{CE}$, $\overline{AB} \cong \overline{CD}$, E is the midpoint of $\overline{BD}$.
PROVE ▶ $\triangle EAB \cong \triangle ECD$



27. **GIVEN** ▶ $\overline{FM} \cong \overline{FN}$, $\overline{DM} \cong \overline{HN}$, $\overline{EF} \cong \overline{GF}$, $\overline{DE} \cong \overline{HG}$
PROVE ▶ $\triangle DEN \cong \triangle HGM$



28. ★ **EXTENDED RESPONSE** When rescuers enter a partially collapsed building they often have to reinforce damaged doors for safety.

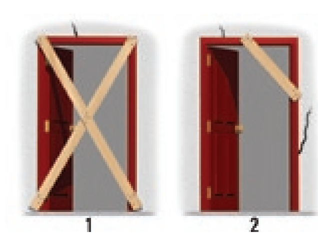
a. Diagonal braces are added to Door 1 as shown below. *Explain* why the door is more stable with the braces.

28a. A figure with diagonal support and fixed side lengths is stable.

b. Would these braces be a good choice for rescuers needing to enter and exit the building through this doorway? **no**

c. In the diagram, Door 2 has only a corner brace. Does this solve the problem from part (b)? **yes**

d. *Explain* why the corner brace makes the door more stable. **It is more stable because there is one diagonal support.**



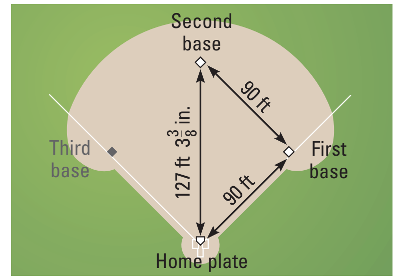
◯ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

27. Statements (Reasons)

1. $\overline{FM} \cong \overline{FN}$, $\overline{DM} \cong \overline{HN}$, $\overline{EF} \cong \overline{GF}$, $\overline{DE} \cong \overline{HG}$ (Given)
2. $MN = NM$ (Reflexive Property of Equality)
3. $FM = FN$, $DM = HN$, $EF = GF$ (Definition of segment congruence)
4. $EF + FN = GF + FN$, $DM + MN = HN + MN$ (Addition Property of Equality)
5. $EF + FN = GF + FM$, $DM + MN = HN + NM$ (Substitution Property of Equality)
6. $EN = GM$, $DN = HM$ (Segment Addition Postulate)
7. $\overline{EN} \cong \overline{GM}$, $\overline{DN} \cong \overline{HM}$ (Definition of segment congruence)
8. $\triangle DEN \cong \triangle HGM$ (SSS)

29. **BASEBALL FIELD** To create a baseball field, start by placing home plate. Then, place second base 127 feet $3\frac{3}{8}$ inches from home plate. Then, you can find first base using two tape measures. Stretch one from second base toward first base and the other from home plate toward first base. The point where the two tape measures cross at the 90 foot mark is first base. You can find third base in a similar manner. *Explain* how and why this process will always work.
Only one triangle can be created from three fixed sides.



C 30. **CHALLENGE** Draw and label the figure described below. Then, identify what is given and write a two-column proof.

In an isosceles triangle, if a segment is added from the vertex between the congruent sides to the midpoint of the third side, then two congruent triangles are formed. **See margin.**

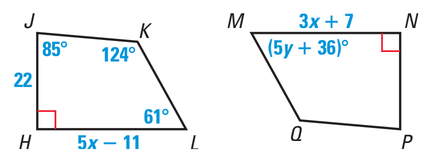
QUIZ

A triangle has the given vertices. Graph the triangle and classify it by its sides. Then determine if it is a right triangle. **1–3. See margin for art.**

1. $A(-3, 0), B(0, 4), C(3, 0)$ **isosceles; not a right triangle**
2. $A(2, -4), B(5, -1), C(2, -1)$ **isosceles; right triangle**
3. $A(-7, 0), B(1, 6), C(-3, 4)$ **scalene; not a right triangle**

In the diagram, $HJKL \cong NPQM$.

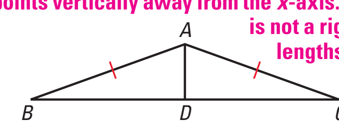
4. Find the value of x. **9**
5. Find the value of y. **5**



6. Consider a transformation in the coordinate plane described by the notation $(x, y) \rightarrow (x + 3, y - 4)$. *Explain* why this is a rigid motion. **6. *Sample answer*: The function rule describes a translation 3 units to the left and 4 units down. A translation is a rigid motion.**
7. Consider a transformation in the coordinate plane described by the notation $(x, y) \rightarrow (x + 1, 3y)$. *Explain* why this is *not* a rigid motion. **7. *Sample answer*: The function rule moves points 1 unit to the right and then stretches points vertically away from the *x*-axis. The transformation is not a rigid motion, because lengths and angles are not preserved.**
8. Write a proof. **See margin.**

GIVEN ▶ $\overline{AB} \cong \overline{AC}$, $\overline{AD}$ bisects $\overline{BC}$.

PROVE ▶ $\triangle ABD \cong \triangle ACD$

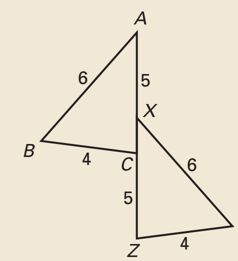


5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

1. The vertices of $\triangle GHI$ and $\triangle RST$ are $G(-2, 5)$, $H(2, 5)$, $I(-2, 2)$, $R(-9, 8)$, $S(-5, 8)$, and $T(-9, 5)$. Is $\triangle GHI \cong \triangle RST$? Explain. **Yes. $GH = RS = 4$, $HI = ST = 5$, and $IG = TR = 3$. By the SSS $\cong$ Post., it follows that $\triangle GHI \cong \triangle RST$.**
2. Is $\triangle ABC \cong \triangle XYZ$? Explain.



Yes. By the Seg. Add. Post., $\overline{AC} \cong \overline{XZ}$. Also, $\overline{AB} \cong \overline{XY}$ and $\overline{BC} \cong \overline{YZ}$. So $\triangle ABC \cong \triangle XYZ$ by the SSS $\cong$ Post.

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

Quiz

An easily-readable reduced copy of the quiz from the Assessment Book can be found at the beginning of this chapter.

30, Quiz 1–3, 8. See Additional Answers.

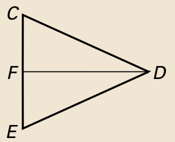
1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

Given: $\overline{DF}$ bisects $\overline{CE}$, $\overline{DC} \cong \overline{DE}$

Prove: $\triangle CDF \cong \triangle EDF$



It is given that $\overline{DC} \cong \overline{DE}$ and $\overline{DF}$ bisects $\overline{CE}$. $\overline{CF} \cong \overline{EF}$ by the def. of bisector. $\overline{DF} \cong \overline{DF}$ by the Refl. Prop. of $\cong$ Segs. So $\triangle CDF \cong \triangle EDF$ by the SSS $\cong$ Post.

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days

Average: 2 days

Advanced: 2 days

Block: 1 block

- See *Teaching Guide/Lesson Plan.*

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 2

How can you use two sides and an angle to prove triangles congruent? **Tell students they will learn how to answer this question by using the SAS Post. and the HL Thm.**

4.5 Prove Triangles Congruent by SAS and HL

Before	You used the SSS Congruence Postulate.
Now	You will use sides and angles to prove congruence.
Why?	So you can show triangles are congruent, as in Ex. 33.

Key Vocabulary
- **leg of a right triangle**
- **hypotenuse**

Consider a relationship involving two sides and the angle they form, their *included* angle. To picture the relationship, form an angle using two pencils.

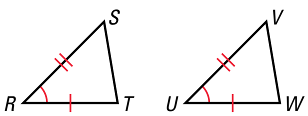
Any time you form an angle of the same measure with the pencils, the side formed by connecting the pencil points will have the same length. In fact, any two triangles formed in this way are congruent.

POSTULATE *For Your Notebook*

POSTULATE 20 Side-Angle-Side (SAS) Congruence Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If	**S**ide	$\overline{RS} \cong \overline{UV}$,
	Angle	$\angle R \cong \angle U$, and
	Side	$\overline{RT} \cong \overline{UW}$,
then		$\triangle RST \cong \triangle UVW$.

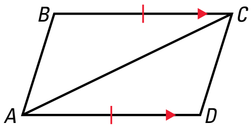


EXAMPLE 1 Use the SAS Congruence Postulate

Write a proof.

GIVEN ▶ $\overline{BC} \cong \overline{DA}$, $\overline{BC} \parallel \overline{AD}$

PROVE ▶ $\triangle ABC \cong \triangle CDA$

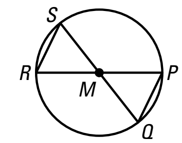


WRITE PROOFS
Make your proof easier to read by identifying the steps where you show congruent sides (S) and angles (A).

	STATEMENTS	REASONS
S	**1.** $\overline{BC} \cong \overline{DA}$	**1.** Given
	2. $\overline{BC} \parallel \overline{AD}$	**2.** Given
A	**3.** $\angle BCA \cong \angle DAC$	**3.** Alternate Interior Angles Theorem
S	**4.** $\overline{AC} \cong \overline{CA}$	**4.** Reflexive Property of Congruence
	5. $\triangle ABC \cong \triangle CDA$	**5.** SAS Congruence Postulate

EXAMPLE 2 Use SAS and properties of shapes

In the diagram, $\overline{QS}$ and $\overline{RP}$ pass through the center M of the circle. What can you conclude about $\triangle MRS$ and $\triangle MPQ$?



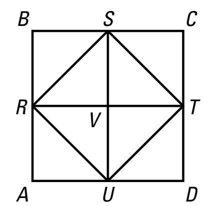
Solution

Because they are vertical angles, $\angle PMQ \cong \angle RMS$. All points on a circle are the same distance from the center, so MP, MQ, MR, and MS are all equal.

▶ $\triangle MRS$ and $\triangle MPQ$ are congruent by the SAS Congruence Postulate.

✓ GUIDED PRACTICE for Examples 1 and 2

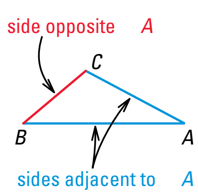
In the diagram, $ABCD$ is a square with four congruent sides and four right angles. R, S, T, and U are the midpoints of the sides of $ABCD$. Also, $\overline{RT} \perp \overline{SU}$ and $\overline{SV} \cong \overline{VU}$.

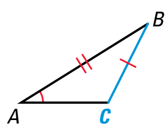


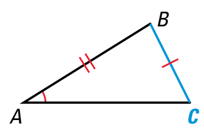
1. Prove that $\triangle SVR \cong \triangle UVR$. **See margin.**
2. Prove that $\triangle BSR \cong \triangle DUT$. **See margin.**

In general, if you know the lengths of two sides and the measure of an angle that is *not included* between them, you can create two different triangles.

READ VOCABULARY
The two sides of a triangle that form an angle are *adjacent* to the angle. The side not adjacent to the angle is *opposite* the angle.

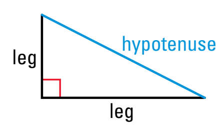






Therefore, SSA is *not* a valid method for proving that triangles are congruent, although there is a special case for right triangles.

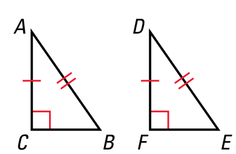
RIGHT TRIANGLES In a right triangle, the sides adjacent to the right angle are called the **legs**. The side opposite the right angle is called the **hypotenuse** of the right triangle.



THEOREM *For Your Notebook*

THEOREM 4.5 Hypotenuse-Leg (HL) Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.



$\triangle ABC \cong \triangle DEF$

Motivating the Lesson

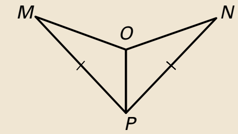
Tell students that in this lesson they will learn what information about angles is need to prove two triangles congruent if it is given that two sides of one triangle are congruent to two sides of another.

3 TEACH

Extra Example 1

Given: $\overline{MP} \cong \overline{NP}$, $\overline{OP}$ bisects $\angle MPN$.

Prove: $\triangle MOP \cong \triangle NOP$



Statements (Reasons)

1. $\overline{MP} \cong \overline{NP}$ (Given)
2. $\overline{OP}$ bisects $\angle MPN$. (Given)
3. $\angle MPO \cong \angle NPO$ (Def. $\angle$ Bis.)
4. $\overline{OP} \cong \overline{OP}$ (Refl. Prop. of $\cong$)
5. $\triangle MOP \cong \triangle NOP$ (SAS $\cong$ Post.)

Extra Example 2

In the diagram R is the center of the circle. If $\angle SRT \cong \angle URT$, what can you conclude about $\triangle SRT$ and $\triangle URT$? **They are $\cong$ by SAS $\cong$ Post.**

Key Question to Ask for Example 2

- Is there only one way to match the vertices to get a true congruence statement? Explain. **No; since the triangles are isosceles, both $\triangle MRS \cong \triangle MQP$ and $\triangle MRS \cong \triangle MPQ$ are true.**

1, 2. See Additional Answers.

Differentiated Instruction

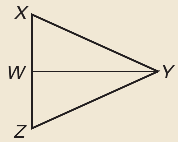
Below Level Put students in pairs so that a weaker student is paired with a better student. Give them fill-in-the-blank proofs to work on. As they complete the proofs, increase the number of blanks that must be filled in. Working in pairs will help the weaker students get the idea of proof and the better students will learn more by explaining the material to their partners. After they have done the fill-in-the-blank proofs, have them move on to completing entire proofs on their own.

See also the *Differentiated Instruction Resources* for more strategies.

Extra Example 3

Given: $\overline{YW} \perp \overline{XZ}$, $\overline{XY} \cong \overline{ZY}$

Prove: $\triangle XYW \cong \triangle ZYW$



Statements (Reasons)

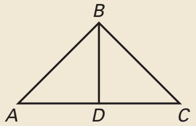
1. $\overline{YW} \perp \overline{XZ}$ (Given)
2. $\overline{XY} \cong \overline{ZY}$ (Given)
3. $\angle XWY$ and $\angle ZWY$ are rt. $\angle$s. ($\perp$ lines form 4 rt. $\angle$s)
4. $\triangle XYW$ and $\triangle ZYW$ are rt. $\triangle$s. (Def. of rt. $\triangle$)
5. $\overline{YW} \cong \overline{YW}$ (Refl. Prop. of $\cong$)
6. $\triangle XYW \cong \triangle ZYW$ (HL $\cong$ Thm.)

Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for **Example 3**. This activity is also part of **Power Presentations**.

Extra Example 4

If you know that $\overline{AB} \cong \overline{CB}$ and $\angle ABD \cong \angle CBD$, what postulate or theorem can you use to conclude that $\triangle ABD$ and $\triangle CBD$, are congruent? **SAS $\cong$ Post.**



Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How can you use two sides and an angle to prove triangles congruent?

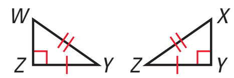
- **Triangles are congruent by the SAS Congruence Postulate.**
- **Right triangles are congruent by the HL Congruence Theorem.**

You can prove triangles congruent if you know that two sides and the included angle of one triangle are congruent to two sides and the included angle of the other. If the triangles are right triangles, you can prove them congruent if they have congruent hypotenuses and a pair of congruent legs.

3, 4. See Additional Answers.

EXAMPLE 3 Use the Hypotenuse-Leg Congruence Theorem

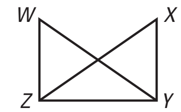
USE DIAGRAMS
If you have trouble matching vertices to letters when you separate the overlapping triangles, leave the triangles in their original orientations.



Write a proof.

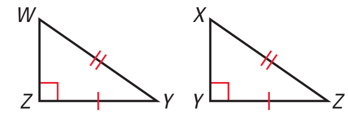
GIVEN ▶ $\overline{WY} \cong \overline{XZ}$, $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$

PROVE ▶ $\triangle WYZ \cong \triangle XZY$



Solution

Redraw the triangles so they are side by side with corresponding parts in the same position. Mark the given information in the diagram.



	STATEMENTS	REASONS
H	1. $\overline{WY} \cong \overline{XZ}$	1. Given
	2. $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$	2. Given
	3. $\angle Z$ and $\angle Y$ are right angles.	3. Definition of $\perp$ lines
	4. $\triangle WYZ$ and $\triangle XZY$ are right triangles.	4. Definition of a right triangle
L	5. $\overline{ZY} \cong \overline{YZ}$	5. Reflexive Property of Congruence
	6. $\triangle WYZ \cong \triangle XZY$	6. HL Congruence Theorem

Animated Geometry at my.hrw.com

EXAMPLE 4 Choose a postulate or theorem

SIGN MAKING You are making a canvas sign to hang on the triangular wall over the door to the barn shown in the picture. You think you can use two identical triangular sheets of canvas. You know that $\overline{RP} \perp \overline{QS}$ and $\overline{PQ} \cong \overline{PS}$. What postulate or theorem can you use to conclude that $\triangle PQR \cong \triangle PSR$?

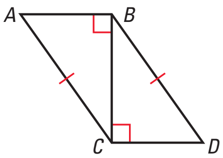
Solution

You are given that $\overline{PQ} \cong \overline{PS}$. By the Reflexive Property, $\overline{RP} \cong \overline{RP}$. By the definition of perpendicular lines, both $\angle RPQ$ and $\angle RPS$ are right angles, so they are congruent. So, two sides and their included angle are congruent.

▶ You can use the SAS Congruence Postulate to conclude that $\triangle PQR \cong \triangle PSR$.

✓ GUIDED PRACTICE for Examples 3 and 4

Use the diagram at the right.



3. Redraw $\triangle ACB$ and $\triangle DBC$ side by side with corresponding parts in the same position. **See margin.**
4. Use the information in the diagram to prove that $\triangle ACB \cong \triangle DBC$. **See margin.**

Differentiated Instruction

Visual Learners Before students begin **Guided Practice Exercises 1 and 2** in previous page, have them trace the diagram. As they work through the problem, have them mark congruent angles and sides on the diagram in corresponding colors. Stress that several sides and angles are congruent to one another.

See also the *Differentiated Instruction Resources* for more strategies.

4.5 EXERCISES

HOMEWORK KEY
○ = See **WORKED-OUT SOLUTIONS** Exs. 13, 19, and 31
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 15, 23, and 39

SKILL PRACTICE

A

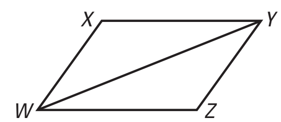
1. **VOCABULARY** Copy and complete: The angle between two sides of a triangle is called the _?_ angle. **included**

2. ★ **WRITING** *Explain* the difference between proving triangles congruent using the SAS and SSS Congruence Postulates. **See margin.**

EXAMPLE 1 for Exs. 3–15

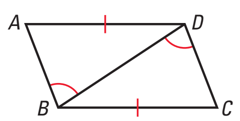
NAMING INCLUDED ANGLES **Use the diagram to name the included angle between the given pair of sides.**

3. $\overline{XY}$ and $\overline{YW}$ **$\angle XYW$**
4. $\overline{WZ}$ and $\overline{ZY}$ **$\angle WZY$**
5. $\overline{ZW}$ and $\overline{YW}$ **$\angle ZWY$**
6. $\overline{WX}$ and $\overline{YX}$ **$\angle WXY$**
7. $\overline{XY}$ and $\overline{YZ}$ **$\angle XYZ$**
8. $\overline{WX}$ and $\overline{WZ}$ **$\angle XWZ$**

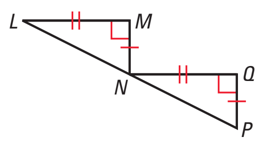


REASONING **Decide whether enough information is given to prove that the triangles are congruent using the SAS Congruence Postulate.**

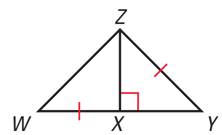
9. $\triangle ABD, \triangle CDB$ **not enough**



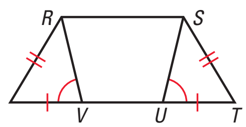
10. $\triangle LMN, \triangle NQP$ **enough**



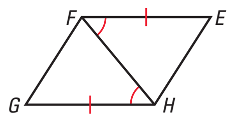
11. $\triangle YXZ, \triangle WXZ$ **not enough**



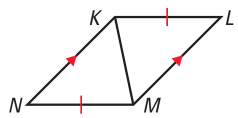
12. $\triangle QRV, \triangle TSU$ **not enough**



13. $\triangle EFH, \triangle GHF$ **enough**



14. $\triangle KLM, \triangle MNK$ **not enough**



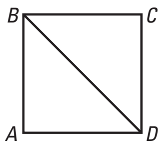
15. ★ **MULTIPLE CHOICE** Which of the following sets of information does not allow you to conclude that $\triangle ABC \cong \triangle DEF$? **B**

(A) $\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}, \angle B \cong \angle E$
(B) $\overline{AB} \cong \overline{DF}, \overline{AC} \cong \overline{DE}, \angle C \cong \angle E$
(C) $\overline{AC} \cong \overline{DF}, \overline{BC} \cong \overline{EF}, \overline{BA} \cong \overline{DE}$
(D) $\overline{AB} \cong \overline{DE}, \overline{AC} \cong \overline{DF}, \angle A \cong \angle D$

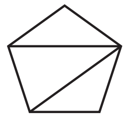
EXAMPLE 2 for Exs. 16–18

APPLYING SAS **In Exercises 16–18, use the given information to name two triangles that are congruent.** ***Explain*** **your reasoning.** **16–18. See margin.**

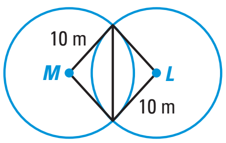
16. *ABCD* is a square with four congruent sides and four congruent angles.



17. *RSTUV* is a regular pentagon.



18. $\overline{MK} \perp \overline{MN}$ and $\overline{KL} \perp \overline{NL}$.



2. SAS requires two sides and the included angle of one triangle to be congruent to the corresponding two sides and the included angle of a second triangle. *SSS* requires that three sides of one triangle be congruent to the corresponding sides of a second triangle.

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–18
Day 2:
Exs. 19–24, 31–36

Average:
Day 1:
Exs. 1, 2, 4–8 even, 9–18, 25–27
Day 2:
Exs. 19–24, 31–39

Advanced:
Day 1:
Exs. 1, 2, 6–8, 12–18, 25–30*
Day 2:
Exs. 19–24, 31–41*

Block:
Exs. 1, 2, 4–8 even, 9–27, 31–39

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 6, 16, 19, 20, 34
Average: 10, 17, 19, 21, 34
Advanced: 12, 18, 19, 22, 34

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

16. $\triangle BAD, \triangle DCB$. *Sample answer:* $\angle A \cong \angle C$ because they are both right angles, $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$ because the sides of a square are congruent, therefore $\triangle BAD \cong \triangle DCB$ by SAS.

17. $\triangle STU, \triangle RVU$; $\overline{ST} \cong \overline{TU} \cong \overline{UV} \cong \overline{VR}$ and $\angle T \cong \angle V$ because it is a regular pentagon, therefore $\triangle STU \cong \triangle RVU$ by SAS.

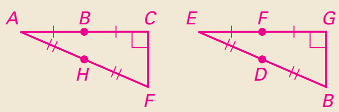
18. $\triangle KMN, \triangle KLN$. *Sample answer:* $\overline{MK} \cong \overline{MN} \cong \overline{LN} \cong \overline{LK}$ since they are all radii of the same size circle. It is given that $\overline{MK} \perp \overline{MN}$ and $\overline{LK} \perp \overline{LN}$. $\angle KMN$ and $\angle KLN$ are right angles and since all right angles are congruent, $\angle KMN \cong \angle KLN$. Therefore $\triangle KMN \cong \triangle KLN$ by SAS.

Avoiding Common Errors

Exercise 12 Students may think that this is an example of the SAS Congruence Postulate. Remind students that SAS requires two sides and the *included* angle.

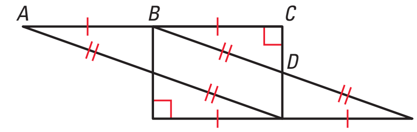
Study Strategy

Exercises 20–21 For each exercise, have students list two sides in one triangle that are congruent to two sides of the other triangle. Then have them ask whether the angles that they know are congruent are the included angles for these sides.

19. 

EXAMPLE 3
for Ex. 19

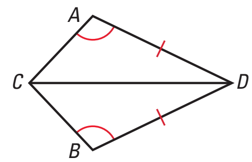
19. **OVERLAPPING TRIANGLES** Redraw $\triangle ACF$ and $\triangle EGB$ so they are side by side with corresponding parts in the same position. *Explain* how you know that $\triangle ACF \cong \triangle EGB$. **HL; see margin for art.**



EXAMPLE 4
for Exs. 20–22

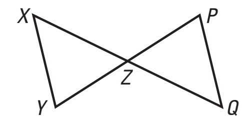
REASONING **Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, state the congruence postulate or theorem you would use.**

20.

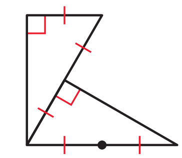


not enough information

21. Z is the midpoint of $\overline{PY}$ and $\overline{XQ}$. **SAS**



22.

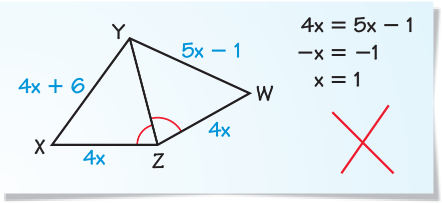


HL

B

23. ★ **WRITING** Suppose both pairs of corresponding legs of two right triangles are congruent. Are the triangles congruent? *Explain*.
Yes; they are congruent by the SAS Congruence Postulate.

24. **ERROR ANALYSIS** *Describe* and correct the error in finding the value of x.
$\overline{YX}$ and $\overline{YW}$ should have the same length since it can be shown that $\triangle XZY \cong \triangle WZY$; $4x + 6 = 5x - 1$, $-x = -7$, $x = 7$.

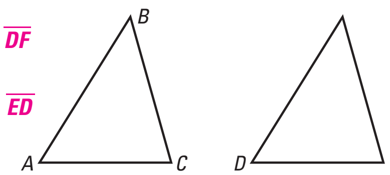


USING DIAGRAMS **In Exercises 25–27, state the third congruence that must be given to prove that $\triangle ABC \cong \triangle DEF$ using the indicated postulate.**

25. **GIVEN** ▶ $\overline{AB} \cong \overline{DE}$, $\overline{CB} \cong \overline{FE}$, $\underline{\ ?\ } \cong \underline{\ ?\ }$
Use the SSS Congruence Postulate. **$\overline{AC} \cong \overline{DF}$**

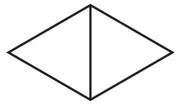
26. **GIVEN** ▶ $\angle A \cong \angle D$, $\overline{CA} \cong \overline{FD}$, $\underline{\ ?\ } \cong \underline{\ ?\ }$
Use the SAS Congruence Postulate. **$\overline{BA} \cong \overline{ED}$**

27. **GIVEN** ▶ $\angle B \cong \angle E$, $\overline{AB} \cong \overline{DE}$, $\underline{\ ?\ } \cong \underline{\ ?\ }$ **$\overline{BC} \cong \overline{EF}$**
Use the SAS Congruence Postulate.



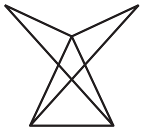
C

28. **USING ISOSCELES TRIANGLES** Suppose $\triangle KLN$ and $\triangle MLN$ are isosceles triangles with $\overline{KL} \cong \overline{LN}$ and $\overline{ML} \cong \overline{LN}$, and $\overline{NL}$ bisects $\angle KLM$. Is there enough information to prove that $\triangle KLN \cong \triangle MLN$? *Explain*.
Yes; they are congruent by SAS.



29. **REASONING** Suppose M is the midpoint of $\overline{PQ}$ in $\triangle PQR$. If $\overline{RM} \perp \overline{PQ}$, *explain* why $\triangle RMP \cong \triangle RMQ$. **Because $\overline{RM} \perp \overline{PQ}$, $\angle RMQ$ and $\angle RMP$ are right angles and thus are congruent. $\overline{QM} \cong \overline{MP}$ and $\overline{MR} \cong \overline{MR}$. So, $\triangle RMP \cong \triangle RMQ$ by SAS.**

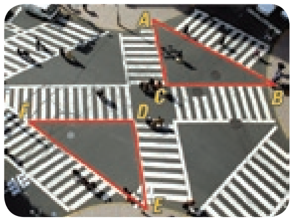
30. **CHALLENGE** Suppose $\overline{AB} \cong \overline{AC}$, $\overline{AD} \cong \overline{AF}$, $\overline{AD} \perp \overline{AB}$, and $\overline{AF} \perp \overline{AC}$. *Explain* why you can conclude that $\triangle ACD \cong \triangle ABF$. **Since $\angle DAC \cong \angle FAB$ the triangles are congruent by SAS.**



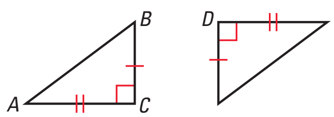
○ = See **WORKED-OUT SOLUTIONS** in Student Resources
★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

A **CONGRUENT TRIANGLES** **In Exercises 31 and 32, identify the theorem or postulate you would use to prove the triangles congruent.**

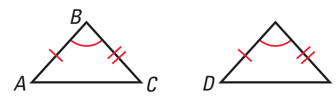
31. 

SAS



32. 

SAS



33. Two sides and the included angle of one sail need to be congruent to two sides and the included angle of the second sail; the two sails need to be right triangles with congruent hypotenuses and one pair of congruent legs.

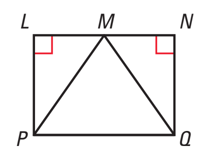
EXAMPLE 3
for Ex. 34

33. SAILBOATS Suppose you have two sailboats. What information do you need to know to prove that the triangular sails are congruent using SAS? using HL?

34. DEVELOPING PROOF Copy and complete the proof.

GIVEN ▶ Point M is the midpoint of $\overline{LN}$.
$\triangle PMQ$ is an isosceles triangle with $\overline{MP} \cong \overline{MQ}$.
$\angle L$ and $\angle N$ are right angles.

PROVE ▶ $\triangle LMP \cong \triangle NMQ$

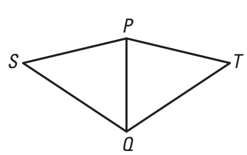


STATEMENTS	REASONS
1. $\angle L$ and $\angle N$ are right angles.	**1.** Given
2. $\triangle LMP$ and $\triangle NMQ$ are right triangles.	**2.** __?__ Definition of a right triangle
3. Point M is the midpoint of $\overline{LN}$.	**3.** __?__ Given
4. __?__ $\overline{LM} \cong \overline{NM}$	**4.** Definition of midpoint
5. $\overline{MP} \cong \overline{MQ}$	**5.** Given
6. $\triangle LMP \cong \triangle NMQ$	**6.** __?__ HL

B **PROOF** **In Exercises 35 and 36, write a proof.** 35, 36. See margin.

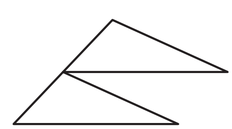
35. GIVEN ▶ $\overline{PQ}$ bisects $\angle SPT$, $\overline{SP} \cong \overline{TP}$

PROVE ▶ $\triangle SPQ \cong \triangle TPQ$



36. GIVEN ▶ $\overline{VX} \cong \overline{XY}$, $\overline{XW} \cong \overline{YZ}$, $\overline{XW} \parallel \overline{YZ}$

PROVE ▶ $\triangle VXW \cong \triangle XYZ$



Reading Strategy

Exercise 34 As students read the given information, have them mark a copy of the diagram to show congruent segments. Students should be able to justify all the marks they make by referring to a piece of given information.

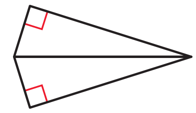
35. Statements (Reasons)
1. $\overline{PQ}$ bisects $\angle SPT$, $\overline{SP} \cong \overline{TP}$. (Given)
2. $\angle SPQ \cong \angle TPQ$ (Definition of angle bisector)
3. $\overline{PQ} \cong \overline{PQ}$ (Reflexive Property of Congruence)
4. $\triangle SPQ \cong \triangle TPQ$ (SAS)

36. Statements (Reasons)
1. $\overline{VX} \cong \overline{XY}$, $\overline{XW} \cong \overline{YZ}$, $\overline{XW} \parallel \overline{YZ}$ (Given)
2. $\angle VXW \cong \angle XYZ$ (Corresponding Angles Postulate)
3. $\triangle VXW \cong \triangle XYZ$ (SAS)

PROOF In Exercises 37 and 38, write a proof. **37, 38. See margin.**

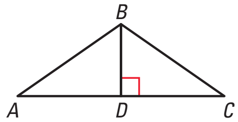
37. **GIVEN** ▶ $\overline{JM} \cong \overline{LM}$

PROVE ▶ $\triangle JKM \cong \triangle LKM$



38. **GIVEN** ▶ D is the midpoint of $\overline{AC}$.

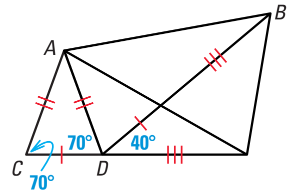
PROVE ▶ $\triangle ABD \cong \triangle CBD$



39. ★ **MULTIPLE CHOICE** Which triangle congruence can you prove, then use to prove that $\angle FED \cong \angle ABF$? **D**

Ⓐ $\triangle ABE \cong \triangle ABF$ Ⓑ $\triangle AED \cong \triangle ABD$

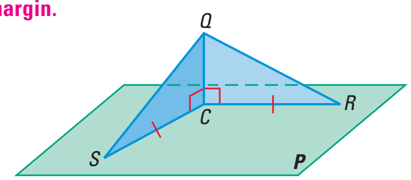
Ⓒ $\triangle ACD \cong \triangle ADF$ Ⓓ $\triangle AEC \cong \triangle ABD$



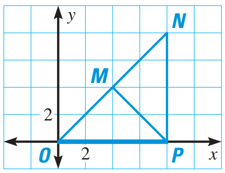
C **40.** **PROOF** Write a two-column proof. **See margin.**

GIVEN ▶ $\overline{CR} \cong \overline{CS}$, $\overline{QC} \perp \overline{CR}$, $\overline{QC} \perp \overline{CS}$

PROVE ▶ $\triangle QCR \cong \triangle QCS$



41. **CHALLENGE** *Describe* how to show that $\triangle PMO \cong \triangle PMN$ using the SSS Congruence Postulate. Then show that the triangles are congruent using the SAS Congruence Postulate without measuring any angles. *Compare* the two methods. **See margin.**



See **EXTRA PRACTICE** in Student Resources **ONLINE QUIZ** at my.hrw.com

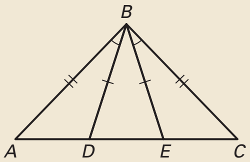
5 ASSESS AND RETEACH

Daily Homework Quiz

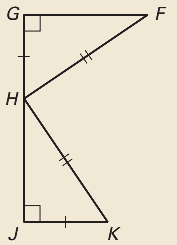
Also available online

Is there enough given information to prove the triangles congruent? If there is, state the postulate or theorem.

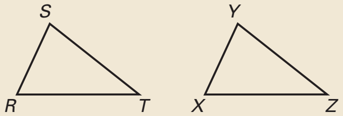
1. $\triangle ABE$, $\triangle CBD$ **SAS ≅ Post.**



2. $\triangle FGH$, $\triangle HJK$ **HL ≅ Thm.**



State a third congruence that would allow you to prove $\triangle RST \cong \triangle XYZ$ by the SAS Congruence Postulate.



3. $\overline{ST} \cong \overline{YZ}$, $\overline{RS} \cong \overline{XY}$ **$\angle S \cong \angle Y$**

4. $\angle T \cong \angle Z$, $\overline{RT} \cong \overline{XZ}$ **$\overline{ST} \cong \overline{YZ}$**

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

37, 38, 40, 41. See Additional Answers.

my.hrw.com
Keystrokes

Investigate Triangles and Congruence

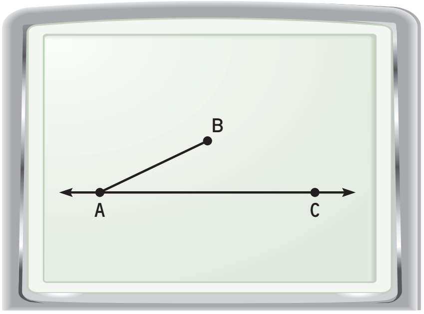
MATERIALS • graphing calculator or computer

QUESTION **Can you prove triangles are congruent by SSA?**

You can use geometry drawing software to show that if two sides and a nonincluded angle of one triangle are congruent to two sides and a nonincluded angle of another triangle, the triangles are not necessarily congruent.

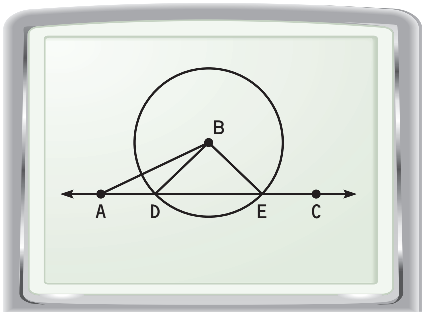
EXAMPLE **Draw two triangles**

STEP 1



Draw a line Draw points A and C. Draw line $\overleftrightarrow{AC}$. Then choose point B so that $\angle BAC$ is acute. Draw $\overline{AB}$.

STEP 2



Draw a circle Draw a circle with center at B so that the circle intersects $\overleftrightarrow{AC}$ at two points. Label the points D and E. Draw $\overline{BD}$ and $\overline{BE}$. Save as "EXAMPLE".

STEP 3 ***Use your drawing***

Explain why $\overline{BD} \cong \overline{BE}$. In $\triangle ABD$ and $\triangle ABE$, what other sides are congruent? What angles are congruent? **They are radii of the same circle; $\overline{AB} \cong \overline{AB}$; $\angle BAD \cong \angle BAE$.**

PRACTICE

1. *Explain* how your drawing shows that $\triangle ABD \not\cong \triangle ABE$. **$\overline{DA}$ is not congruent to $\overline{EA}$.**
2. Change the diameter of your circle so that it intersects $\overleftrightarrow{AC}$ in only one point. Measure $\angle BDA$. *Explain* why there is exactly one triangle you can draw with the measures AB, BD, and a 90° angle at $\angle BDA$. **2. Since $\triangle ABD$ is a right triangle, the Hypotenuse-Leg Congruence Theorem guarantees that any triangle with these dimensions will also be congruent to $\triangle ABD$.**
3. *Explain* why your results show that SSA cannot be used to show that two triangles are congruent but that HL can. **The activity shows that SSA can yield two non-congruent triangles but that HL results in only one triangle.**

1 PLAN AND PREPARE

Learn the Method

- Students will use geometry software to show that SSA does not necessarily prove triangles congruent.

Keystroke Help

Keystrokes for several models of calculators are available in blackline format in the *Chapter Resource Book*.

2 TEACH

Tips for Success

Be sure students make $\angle BAC$ acute and draw the circle with a small enough radius so it intersects $\overleftrightarrow{AC}$ in two points on the same side of point A. Have students hide point C so they do not use it as a vertex of their triangle.

Extra Example

Draw a line, label points A and C on the line, and construct an obtuse $\angle CAB$. Hide point C. Construct a circle with center B that intersects $\overline{AC}$. How many triangles are formed with vertices A, B, and the intersecton of the circle with $\overleftrightarrow{AC}$? Are they congruent? **2; no**

3 ASSESS AND RETEACH

1. Suppose two sides and an angle of one triangle are congruent to two sides and the corresponding angle of another triangle. What do you need to know about the angles to be sure that the triangles are congruent? **The angles must be the included angles for the congruent sides or the given congruent angles must be right angles.**

1a. $\triangle ACE$ and $\triangle DCF$ are obtuse triangles, and $\triangle ECB$, $\triangle FCG$, $\triangle ABC$, and $\triangle DGC$ are acute triangles.

2. *Sample answer:* Using the distance formula it can be shown that $\triangle PQR \cong \triangle STR$ by SSS.

4. Yes; Because $\overline{AC} \cong \overline{GE}$ and $\overline{AB} \cong \overline{FE}$, $\overline{BC} \cong \overline{GF}$. Also, since $\overline{AG} \cong \overline{CE}$ and $\overline{AH} \cong \overline{DE}$, $\overline{HG} \cong \overline{CD}$. $\angle G$ and $\angle C$ are right angles, so they are congruent. Therefore, $\triangle BCD \cong \triangle FGH$ by SAS.

5a. Statements (Reasons)

1. $\overline{BG} \perp \overline{FH}$, $\overline{GF} \cong \overline{GH}$ (Given)
2. $\angle BGF$ and $\angle BGH$ are right angles. (Definition of perpendicular lines)
3. $\angle BGF \cong \angle BGH$ (Right Angles Congruence Theorem)
4. $\overline{BG} \cong \overline{BG}$ (Reflexive Property of Congruence)
5. $\triangle FGB \cong \triangle HGB$ (SAS)

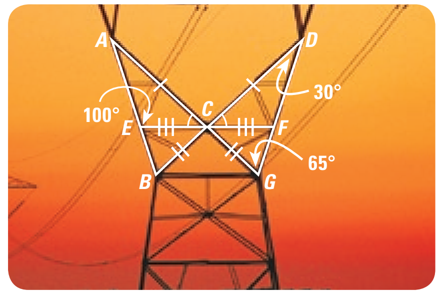
5b. yes;
Statements (Reasons)

1. $\overline{DF} \cong \overline{EH}$, $m\angle EHB = 25°$, $m\angle BFG = 65°$, $\overline{DF} \perp \overline{AG}$ at point F (Given)
2. $\triangle FGB \cong \triangle HGB$ (Problem 5a)
3. $\overline{FB} \cong \overline{HB}$ (Corr. parts of $\cong$ $\triangle$s are $\cong$.)
4. $\angle DFG$ is a right angle. (Definition of perpendicular lines)
5. $m\angle DFG = 90°$ (Definition of right angle)
6. $m\angle DFB + m\angle BFG = m\angle DFC$ (Angle Addition Postulate)
7. $m\angle DFB + 65° = 90°$ (Substitution Property of Equality)
8. $m\angle DFB = 25°$ (Subtraction Property of Equality)
9. $m\angle DFB = m\angle EHB$ (Transitive Property of Equality)
10. $\angle DFB \cong \angle EHB$ (Definition of congruent angles)
11. $\triangle BDF \cong \triangle BEH$ (SAS)

MIXED REVIEW of Problem Solving

STATE TEST PRACTICE
my.hrw.com

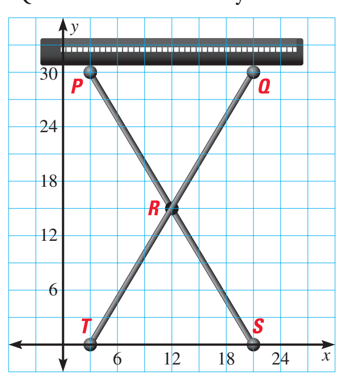
1. MULTI-STEP PROBLEM In the diagram, $\overline{AC} \cong \overline{CD}$, $\overline{BC} \cong \overline{CG}$, $\overline{EC} \cong \overline{CF}$, and $\angle ACE \cong \angle DCF$.



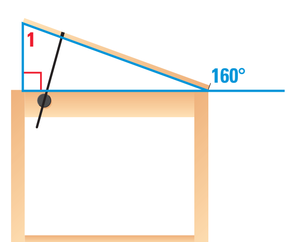
a. Classify each triangle in the figure by angles. **See margin.**

b. Classify each triangle in the figure by sides. **All triangles are scalene.**

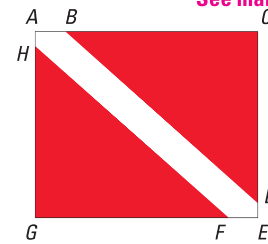
2. OPEN-ENDED *Explain* how you know that $\triangle PQR \cong \triangle STR$ in the keyboard stand shown. **See margin.**



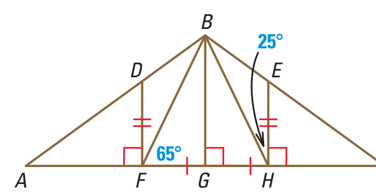
3. GRIDDED ANSWER In the diagram below, find the measure of $\angle 1$ in degrees. **70°**



4. SHORT RESPONSE A rectangular "diver down" flag is used to indicate that scuba divers are in the water. On the flag, $\overline{AB} \cong \overline{FE}$, $\overline{AH} \cong \overline{DE}$, $\overline{CE} \cong \overline{AG}$, and $\overline{EG} \cong \overline{AC}$. Also, $\angle A$, $\angle C$, $\angle E$, and $\angle G$ are right angles. Is $\triangle BCD \cong \triangle FGH$? *Explain.* **See margin.**



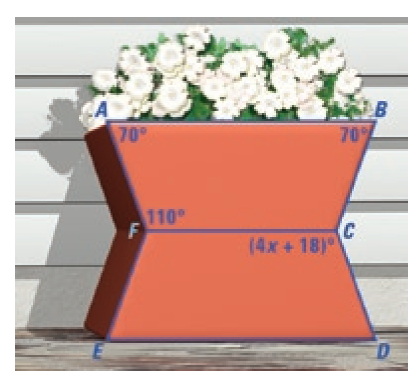
5. EXTENDED RESPONSE A roof truss is a network of pieces of wood that forms a stable structure to support a roof, as shown below.



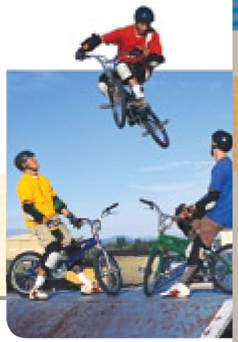
a. Prove that $\triangle FGB \cong \triangle HGB$. **a, b. See margin.**

b. Is $\triangle BDF \cong \triangle BEH$? If so, prove it.

6. GRIDDED ANSWER In the diagram below, $BAFC \cong DEFC$. Find the value of x. **23**



4.6 Prove Triangles Congruent by ASA and AAS



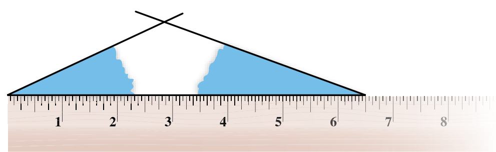
Before You used the SSS, SAS, and HL congruence methods.

Now You will use two more methods to prove congruences.

Why? So you can recognize congruent triangles in bikes, as in Exs. 23–24.

Key Vocabulary
- flow proof

Suppose you tear two angles out of a piece of paper and place them at a fixed distance on a ruler. Can you form more than one triangle with a given length and two given angle measures as shown below?



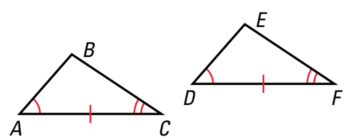
In a polygon, the side connecting the vertices of two angles is the *included* side. Given two angle measures and the length of the included side, you can make only one triangle. So, all triangles with those measurements are congruent.

THEOREMS *For Your Notebook*

POSTULATE 21 Angle-Side-Angle (ASA) Congruence Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

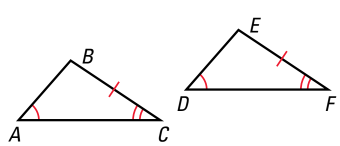
If **A**ngle $\angle A \cong \angle D$,
Side $\overline{AC} \cong \overline{DF}$, and
Angle $\angle C \cong \angle F$,
then $\triangle ABC \cong \triangle DEF$.



THEOREM 4.6 Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

If **A**ngle $\angle A \cong \angle D$,
Angle $\angle C \cong \angle F$, and
Side $\overline{BC} \cong \overline{EF}$,
then $\triangle ABC \cong \triangle DEF$.

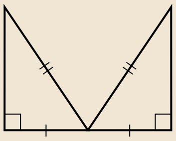


1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

Tell whether the pair of triangles is congruent or not and why.



Yes; HL ≅ Thm.

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days
Average: 2 days
Advanced: 2 days
Block: 1 block
- See *Teaching Guide/Lesson Plan.*

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 2

If a side of one triangle is congruent to a side of another triangle, what information about the angles would allow you to prove the triangles are congruent? **Tell students they will learn how to answer this question by learning about the ASA Postulate and the AAS Theorem.**

Motivating the Lesson

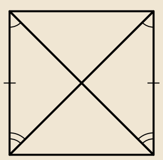
Tell students that in this lesson they will learn how a surveyor can decide whether two triangular plots of land are the same size and shape by measuring the length of one side and two angles of each triangle.

3 TEACH

Extra Example 1

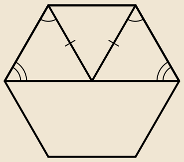
Can the triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

a.



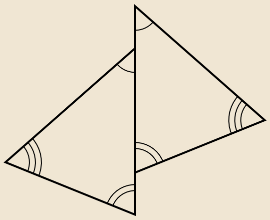
ASA Congruence Postulate

b.



AAS Congruence Theorem

c.



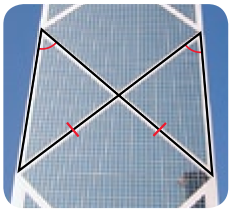
cannot be proven congruent

2. See Additional Answers.

EXAMPLE 1 Identify congruent triangles

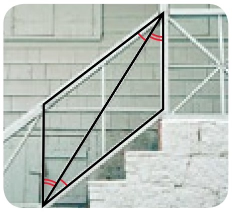
Can the triangles be proven congruent with the information given in the diagram? If so, state the postulate or theorem you would use.

a.



b.

c.



Solution

a. The vertical angles are congruent, so two pairs of angles and a pair of non-included sides are congruent. The triangles are congruent by the AAS Congruence Theorem.

b. There is not enough information to prove the triangles are congruent, because no sides are known to be congruent.

c. Two pairs of angles and their included sides are congruent. The triangles are congruent by the ASA Congruence Postulate.

AVOID ERRORS
You need at least one pair of congruent corresponding sides to prove two triangles congruent.

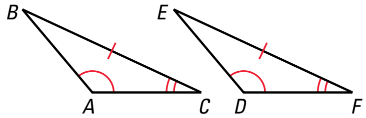
FLOW PROOFS You have written two-column proofs and paragraph proofs. A **flow proof** uses arrows to show the flow of a logical argument. Each reason is written below the statement it justifies.

EXAMPLE 2 Prove the AAS Congruence Theorem

Prove the Angle-Angle-Side Congruence Theorem.

GIVEN ▶ $\angle A \cong \angle D$, $\angle C \cong \angle F$, $\overline{BC} \cong \overline{EF}$

PROVE ▶ $\triangle ABC \cong \triangle DEF$



$\angle A \cong \angle D$ → $\angle B \cong \angle E$
Given / Third ∡ Thm.

$\angle C \cong \angle F$
Given

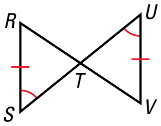
$\overline{BC} \cong \overline{EF}$
Given

$\triangle ABC \cong \triangle DEF$
ASA Congruence Post.

Animated Geometry at my.hrw.com

✓ GUIDED PRACTICE for Examples 1 and 2

1. In the diagram at the right, what postulate or theorem can you use to prove that $\triangle RST \cong \triangle VUT$? *Explain.*
AAS; $\angle RTS$ and $\angle VTU$ are congruent because they are vertical angles.



2. Rewrite the proof of the Triangle Sum Theorem, learned in the lesson *Apply Triangle Sum Properties*, as a flow proof.
See margin.

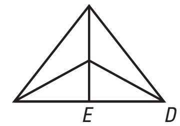
Differentiated Instruction

Inclusion It may be challenging for some students to remember the triangle congruence postulates. Point out that the order in which the letters in the abbreviations appear is the same order in which the corresponding parts must exist in the two triangles. Write SSS, SAS, HL, ASA, and AAS on the board and have students describe what corresponding parts must be congruent for each postulate.

See also the *Differentiated Instruction Resources* for more strategies.

EXAMPLE 3 Write a flow proof

In the diagram, $\overline{CE} \perp \overline{BD}$ and $\angle CAB \cong \angle CAD$. Write a flow proof to show $\triangle ABE \cong \triangle ADE$.



Solution

GIVEN ▶ $\overline{CE} \perp \overline{BD}$, $\angle CAB \cong \angle CAD$

PROVE ▶ $\triangle ABE \cong \triangle ADE$

$\angle CAB \cong \angle CAD$ — Given

$\angle BAE$ and $\angle CAB$ are supplementary. $\angle DAE$ and $\angle CAD$ are supplementary. — Linear Pair Postulate

$\overline{CE} \perp \overline{BD}$ — Given

$\angle BAE \cong \angle DAE$ — Congruent Supps. Thm.

$\overline{AE} \cong \overline{AE}$ — Reflexive Prop.

$m\angle AEB = m\angle AED = 90°$ — Def. of $\perp$ lines

$\angle AEB \cong \angle AED$ — All right $\angle$s are $\cong$.

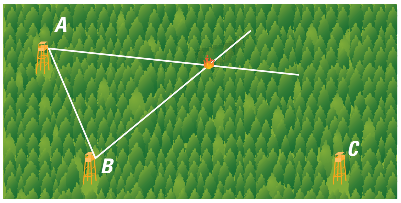
$\triangle ABE \cong \triangle ADE$ — ASA Congruence Post.

EXAMPLE 4 Standardized Test Practice

FIRE TOWERS The forestry service uses fire tower lookouts to watch for forest fires. When the lookouts spot a fire, they measure the angle of their view and radio a dispatcher. The dispatcher then uses the angles to locate the fire. How many lookouts are needed to locate a fire?

(A) 1 (B) 2 (C) 3 (D) Not enough information

The locations of tower A, tower B, and the fire form a triangle. The dispatcher knows the distance from tower A to tower B and the measures of $\angle A$ and $\angle B$. So, the measures of two angles and an included side of the triangle are known.



By the ASA Congruence Postulate, all triangles with these measures are congruent. So, the triangle formed is unique and the fire location is given by the third vertex. Two lookouts are needed to locate the fire.

▶ The correct answer is B. (A) (B) (C) (D)

GUIDED PRACTICE for Examples 3 and 4

3. In Example 3, suppose $\angle ABE \cong \angle ADE$ is also given. What theorem or postulate besides ASA can you use to prove that $\triangle ABE \cong \triangle ADE$? **AAS Congruence Theorem**

4. WHAT IF? In Example 4, suppose a fire occurs directly between tower B and tower C. Could towers B and C be used to locate the fire? *Explain.* **No; no triangle is formed by the location of the fire and the towers, so the fire could be anywhere between towers B and C.**

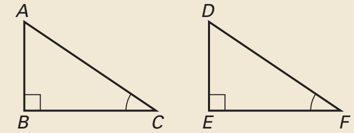
Differentiated Instruction

Advanced Have the students construct a triangle given two side lengths and the measure of the included angle. Also have them construct a triangle given the measures of two angles and the length of the included side. Challenge them to construct a triangle given the measures of two angles and the length of a non-included side and then the lengths of two sides and measure of a non-included acute angle.

See also the *Differentiated Instruction Resources* for more strategies.

Extra Example 2

In the diagram, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AC} \cong \overline{DF}$, and $\angle C \cong \angle F$. Prove that $\triangle ABC \cong \triangle DEF$.



$\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$ — Given

$\angle B$ is a right $\angle$, $\angle E$ is a right $\angle$ — Def. of $\perp$ lines

$\angle B \cong \angle E$ — Right $\angle$ Cong. Thm.

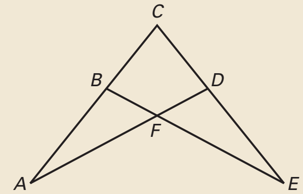
$\angle C \cong \angle F$ — Given

$\overline{AC} \cong \overline{DF}$ — Given

$\triangle ABC \cong \triangle DEF$ — AAS Cong. Post.

Extra Example 3

In the diagram, $\angle CBF \cong \angle CDF$ and $\overline{BF} \cong \overline{FD}$. Write a flow proof to show that $\triangle ABF \cong \triangle EDF$.



$\angle CBF \cong \angle CDF$ — Given

$\angle CBF$, $\angle ABF$ supplementary; $\angle CDF$, $\angle EDF$ supplementary — Linear Pair Post.

$\angle ABF \cong \angle EDF$ — Cong. Supps. Thm.

$\overline{BF} \cong \overline{FD}$ — Given

$\angle BFA \cong \angle DFE$ — Vert. $\angle$s Thm.

$\triangle ABF \cong \triangle EDF$ — ASA $\cong$ Post.

Extra Example 4

Several observers along a straight shoreline measure the angle between the shoreline and the parachute of a space capsule that has just returned to Earth. If the distance between each pair of observers is known, how many of the angle measures are needed to locate the capsule parachute? **two**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: If a side of one triangle is congruent to a side of another triangle, what information about the angles would allow you to prove the triangles are congruent?

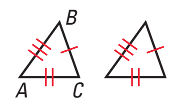
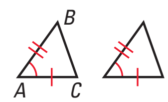
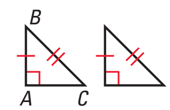
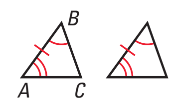
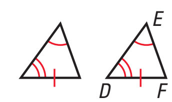
- **Triangles are congruent by the ASA Congruence Postulate.**
- **Triangles are congruent by the AAS Congruence Theorem.**
- **Another format for proofs is the flow proof.**

The triangles will be congruent if the conditions of the ASA Congruence Postulate or of the AAS Congruence Theorem are met.

CONCEPT SUMMARY — *For Your Notebook*

Triangle Congruence Postulates and Theorems

You have learned five methods for proving that triangles are congruent.

SSS	SAS	HL (right △s only)	ASA	AAS
				
All three sides are congruent.	Two sides and the included angle are congruent.	The hypotenuse and one of the legs are congruent.	Two angles and the included side are congruent.	Two angles and a (non-included) side are congruent.

In the Exercises, you will prove three additional theorems about the congruence of right triangles: **A**ngle-**L**eg, **L**eg-**L**eg, and **H**ypotenuse-**A**ngle.

4.6 EXERCISES

HOMEWORK KEY
○ = See **WORKED-OUT SOLUTIONS** Exs. 5, 9, and 27
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 7, 21, and 26

SKILL PRACTICE

A

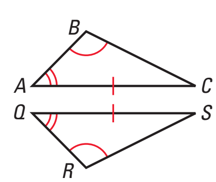
1. **VOCABULARY** Name one advantage of using a flow proof rather than a two-column proof. ***Sample answer:*** **A flow proof shows the flow of a logical argument.**

2. ★ **WRITING** You know that a pair of triangles has two pairs of congruent corresponding angles. What other information do you need to show that the triangles are congruent? **a pair of congruent corresponding sides that are either both included or both not included**

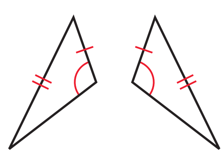
EXAMPLE 1 for Exs. 3–7

IDENTIFY CONGRUENT TRIANGLES **Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem you would use.**

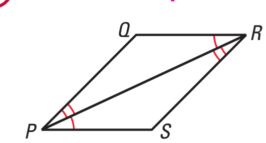
3. △*ABC*, △*QRS* **yes; AAS**



4. △*XYZ*, △*JKL* **no**

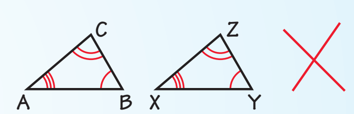


5. △*PQR*, △*RSP* **yes; ASA**



6. **ERROR ANALYSIS** *Describe* the error in concluding that △*ABC* ≅ △*XYZ*. **There is no AAA postulate or theorem.**

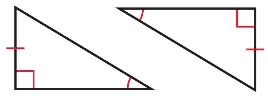
By AAA,
△ABC ≅ △XYZ.



7. ★ **MULTIPLE CHOICE** Which postulate or theorem can you use to prove that $\triangle ABC \cong \triangle HJK$? **B**

(A) HL (B) AAS

(C) SAS (D) Not enough information



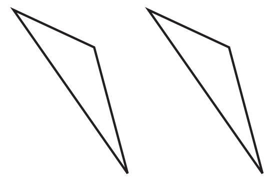
EXAMPLE 2 for Exs. 8–13

DEVELOPING PROOF **State the third congruence that is needed to prove that $\triangle FGH \cong \triangle LMN$ using the given postulate or theorem.**

8. **GIVEN** ▶ $\overline{GH} \cong \overline{MN}$, $\angle G \cong \angle M$, _?_ $\cong$ _?_
Use the AAS Congruence Theorem. **$\angle F$, $\angle L$**

9. **GIVEN** ▶ $\overline{FG} \cong \overline{LM}$, $\angle G \cong \angle M$, _?_ $\cong$ _?_
Use the ASA Congruence Postulate. **$\angle F$, $\angle L$**

10. **GIVEN** ▶ $\overline{FH} \cong \overline{LN}$, $\angle H \cong \angle N$, _?_ $\cong$ _?_
Use the SAS Congruence Postulate. **$\overline{HG}$, $\overline{NM}$**

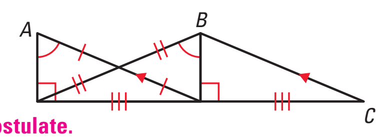


OVERLAPPING TRIANGLES ***Explain*** **how you can prove that the indicated triangles are congruent using the given postulate or theorem.** **11, 12. See margin**

11. $\triangle AFE \cong \triangle DFB$ by SAS

12. $\triangle AED \cong \triangle BDE$ by AAS

13. $\triangle AED \cong \triangle BDC$ by ASA
$\angle EDA \cong \angle DCB$ by the Corresponding Angles Postulate.



B **DETERMINING CONGRUENCE** **Tell whether you can use the given information to determine whether $\triangle ABC \cong \triangle DEF$.** ***Explain*** **your reasoning.**

14. $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$ **yes; SAS**

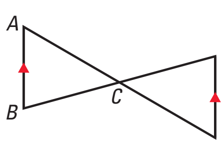
15. $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

16. $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AC} \cong \overline{DE}$
No; $\overline{AC}$ and $\overline{DE}$ are not corresponding sides.

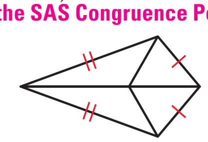
17. $\overline{AB} \cong \overline{EF}$, $\overline{BC} \cong \overline{FD}$, $\overline{AC} \cong \overline{DE}$

IDENTIFY CONGRUENT TRIANGLES **Is it possible to prove that the triangles are congruent? If so, state the postulate(s) or theorem(s) you would use.**

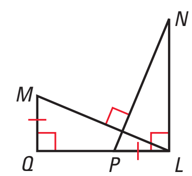
18. $\triangle ABC$, $\triangle DEC$ **no**



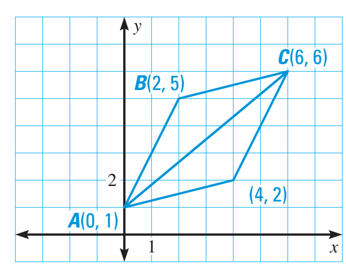
19. $\triangle TUV$, $\triangle TWV$
yes; the SAS Congruence Postulate



20. $\triangle QML$, $\triangle LPN$ **no**



21. ★ **EXTENDED RESPONSE** Use the graph at the right. **a–c. See margin.**
a. Show that $\angle CAD \cong \angle ACB$. *Explain* your reasoning.
b. Show that $\angle ACD \cong \angle CAB$. *Explain* your reasoning.
c. Show that $\triangle ABC \cong \triangle CDA$. *Explain* your reasoning.



C 22. **CHALLENGE** Use a coordinate plane.
a. Graph the lines $y = 2x + 5$, $y = 2x - 3$, and $x = 0$ in the same coordinate plane. **See margin.**
b. Consider the equation $y = mx + 1$. For what values of m will the graph of the equation form two triangles if added to your graph? For what values of m will those triangles be congruent right triangles? *Explain*.

11. $\angle AFE \cong \angle DFB$ by the Vertical Angles Theorem.

12. $\overline{ED} \cong \overline{ED}$ by the Reflexive Property of Segment Congruence.

15. No; there is no AAA postulate or theorem.

17. No; the segments that are congruent are not corresponding sides.

22b. Any real value except 2; $-\frac{1}{2}$ or 0; the resulting triangles are congruent by AAS or ASA.

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–13
Day 2:
Exs. 14–17, 23–30

Average:
Day 1:
Exs. 1, 2, 4–7, 9–13, 18–20
Day 2:
Exs. 14–17, 21, 23–34

Advanced:
Day 1:
Exs. 1, 2, 5–7, 9–13, 18–20, 22*
Day 2:
Exs. 14–17, 23–35*

Block:
Exs. 1, 2, 4–7, 9–21, 23–34

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 3, 8, 14, 25, 26
Average: 4, 10, 15, 25, 26
Advanced: 5, 12, 16, 25, 26

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

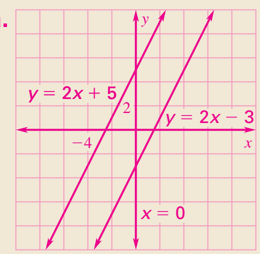
An easily-readable reduced practice page can be found at the beginning of this chapter.

21a. $\overline{BC}$ and $\overline{AD}$ are parallel, because their slopes are equal, with $\overline{AC}$ being a transversal. The Alternate Interior Angles Theorem applies.

21b. $\overline{AB}$ and $\overline{CD}$ are parallel, because their slopes are equal, with $\overline{AC}$ being a transversal. The Alternate Interior Angles Theorem applies.

21c. Using parts 21a, 21b, and the fact that $\overline{AC} \cong \overline{CA}$, they are congruent by ASA.

22a.



Avoiding Common Errors

Exercise 7 Some students may think that they must find some way to use the HL Congruence Theorem, since the triangles are right triangles. Point out that that the problem gives two pairs of congruent angles but only one pair of congruent sides. This suggests that it might be necessary to use the ASA Congruence Postulate or the AAS Congruence Theorem.

Study Strategy

Exercise 19 Students may notice that there is more than one way to conclude that $\triangle TUV \cong \triangle TWV$. Students may want to write a brief explanation of why they chose a particular postulate or theorem.

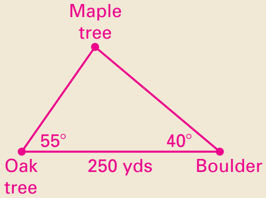
Mathematical Reasoning

Exercise 25 Ask students how else the triangles could be proved congruent.

Internet Reference

Exercise 26 For more information about orienteering, visit the International Orienteering Federation's site at www.orienteering.org

26.



Yes. *Sample answer:* The triangle formed with these measures is unique and the third vertex gives the location of the maple tree.

PROBLEM SOLVING

23. Two pairs of angles and an included pair of sides are congruent. The triangles are congruent by ASA.

24. Two pairs of angles and a nonincluded pair of sides are congruent. The triangles are congruent by AAS.

A

CONGRUENCE IN BICYCLES *Explain* why the triangles are congruent. **23, 24. See margin.**

23.

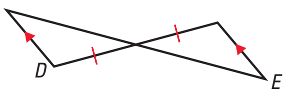
24.

EXAMPLE 3 for Ex. 25

25. FLOW PROOF Copy and complete the flow proof.

GIVEN ▶ $\overline{AD} \parallel \overline{CE}$, $\overline{BD} \cong \overline{BC}$

PROVE ▶ $\triangle ABD \cong \triangle EBC$



$\overline{AD} \parallel \overline{CE}$ (**?**) — Given

$\angle A \cong \angle E$ — **?** Alt. Int. $\angle$s Thm.

$\angle C \cong \angle D$ — **?** Alt. Int. $\angle$s Thm.

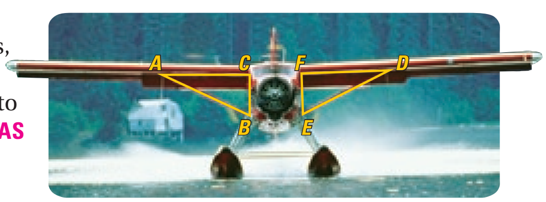
$\overline{BD} \cong \overline{BC}$ — **?** Given

$\triangle ABD \cong \triangle EBC$ — **?** AAS

EXAMPLE 4 for Ex. 26

26. ★ SHORT RESPONSE You are making a map for an orienteering race. Participants start at a large oak tree, find a boulder 250 yards due east of the oak tree, and then find a maple tree that is 50° west of north of the boulder and 35° east of north of the oak tree. Sketch a map. Can you locate the maple tree? *Explain*. **See margin.**

27. AIRPLANE In the airplane at the right, $\angle C$ and $\angle F$ are right angles, $\overline{BC} \cong \overline{EF}$, and $\angle A \cong \angle D$. What postulate or theorem allows you to conclude that $\triangle ABC \cong \triangle DEF$? **AAS**



28. Since all right angles are congruent and the right angles are the included angles of the congruent legs in the triangles, the triangles are congruent by SAS.

29. Since all right angles are congruent, the two triangles are congruent by either AAS, if the side is not included, or ASA, if it is the included side.

B

RIGHT TRIANGLES **In the lesson *Prove Triangles Congruent by SAS and HL*, you learned the Hypotenuse-Leg Theorem for right triangles. In Exercises 28–30, write a paragraph proof for these other theorems about right triangles.**

28. Leg-Leg (LL) Theorem If the legs of two right triangles are congruent, then the triangles are congruent.

29. Angle-Leg (AL) Theorem If an angle and a leg of a right triangle are congruent to an angle and a leg of a second right triangle, then the triangles are congruent.

30. Hypotenuse-Angle (HA) Theorem If an angle and the hypotenuse of a right triangle are congruent to an angle and the hypotenuse of a second right triangle, then the triangles are congruent. **Since all right angles are congruent, the two triangles are congruent by AAS.**

○ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

Michael Melford/Getty Images

31. Statements (Reasons)

1. $\overline{AK} \cong \overline{CJ}$, $\angle BJK \cong \angle BKJ$, $\angle A \cong \angle C$ (Given)

2. $\triangle ABK \cong \triangle CBJ$ (ASA)

32.

$\overline{VW} \cong \overline{UW}$, $\angle X \cong \angle Z$ — Given

$\angle W \cong \angle W$ — Reflexive Property of Congruence

→ $\triangle XWV \cong \triangle ZWU$ — AAS

33.

$\angle NKM \cong \angle LMK$, $\angle L \cong \angle N$ — Given

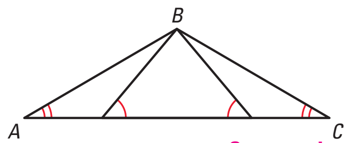
$\overline{KM} \cong \overline{MK}$ — Reflexive Prop. of Congruence

→ $\triangle NMK \cong \triangle LKM$ — AAS

31. PROOF Write a two-column proof.

GIVEN ▶ $\overline{AK} \cong \overline{CJ}$, $\angle BJK \cong \angle BKJ$, $\angle A \cong \angle C$

PROVE ▶ $\triangle ABK \cong \triangle CBJ$

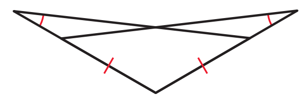


See margin.

32. PROOF Write a flow proof.

GIVEN ▶ $\overline{VW} \cong \overline{UW}$, $\angle X \cong \angle Z$

PROVE ▶ $\triangle XWV \cong \triangle ZWU$

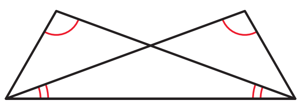


See margin.

33. PROOF Write a proof.

GIVEN ▶ $\angle NKM \cong \angle LMK$, $\angle L \cong \angle N$

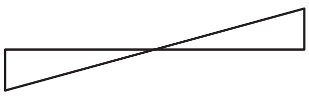
PROVE ▶ $\triangle NMK \cong \triangle LKM$ **See margin.**



34. PROOF Write a proof.

GIVEN ▶ X is the midpoint of $\overline{VY}$ and $\overline{WZ}$.

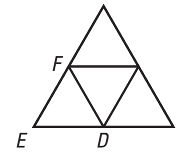
PROVE ▶ $\triangle VWX \cong \triangle YZX$ **See margin.**



C **35. CHALLENGE** Write a proof.

GIVEN ▶ $\triangle ABF \cong \triangle DFB$, F is the midpoint of $\overline{AE}$, B is the midpoint of $\overline{AC}$.

PROVE ▶ $\triangle FDE \cong \triangle BCD \cong \triangle ABF$ **See margin.**

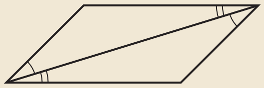


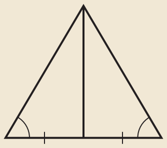
5 ASSESS AND RETEACH

Daily Homework Quiz

Also available online

Tell whether each pair of triangles is congruent by SAS, ASA, SSS, AAS, or HL. If it is not possible to prove the triangles congruent, write *not necessarily congruent*.

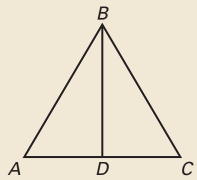
1.  **ASA**

2.  **not necessarily congruent**

3. Write a flow proof.

Given: $\overline{BD}$ bisects $\angle ABC$, $\angle A \cong \angle C$

Prove: $\triangle ABD \cong \triangle CBD$



$\overline{BD}$ bisects $\angle ABC$. Given

$\angle A \cong \angle C$ Given

$\angle ABD \cong \angle CBD$ Def. of $\angle$ Bisector

$\overline{BD} \cong \overline{BD}$ Reflex. Prop. of $\cong$ Segs.

$\triangle ABD \cong \triangle CBD$ AAS $\cong$ Thm.

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

34, 35. See Additional Answers.

1 PLAN AND PREPARE

Explore the Concept

- Students will construct congruent triangles given three segments, or given two segments and an included angle.
- Students will discover that their work is completely symmetric, and this may help them see that reflection is a rigid motion for mapping the resulting triangles to each other.

Materials

Each student will need:

- compass
- straightedge

Recommended Time

Work activity: 15 min

Discuss results: 5 min

Grouping

Students should work individually.

2 TEACH

Tips for Success

Before beginning Explore 1, make sure students are familiar with the construction for copying a segment. Also, point out that it is easiest if students use three different lengths. Stress that if two of the segments are the same length, then only two congruent (isosceles) triangles will be created. Similarly, if all three segments are the same length, only two congruent (equilateral) triangles result.

Alternative Strategy

Ask students to try Explore 1 using the shortest segment as their first segment. The resulting drawings will look different than those shown, but four congruent triangles will still be created.

Rigid Motions and Congruence

MATERIALS • compass • straightedge

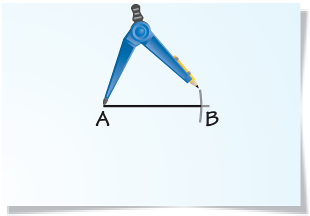
QUESTION **How does triangle congruence follow from rigid motions?**

In the following constructions, rigid motions will be used to explain the criteria for triangle congruence.

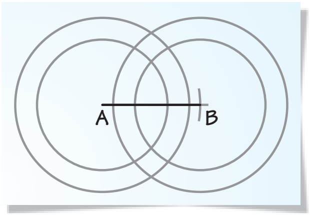
EXPLORE 1 **Construct triangles from three segments (SSS)**

Given three segments, like the ones at the right, use these steps to construct triangles with those three side lengths.

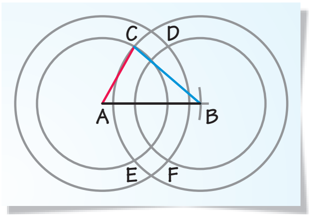
STEP 1 ***Copy a segment*** Copy one of the line segments. In the diagram, the longest segment is copied. Label the endpoints *A* and *B*.



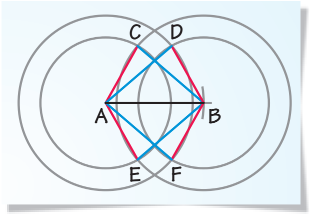
STEP 2 ***Draw circles*** At both endpoints of the segment, draw circles using the other two given side lengths as radii.



STEP 3 ***Find vertices*** Identify the four places where circles of different radii intersect. Label the points *C*, *D*, *E*, and *F*.

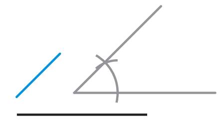


STEP 4 ***Draw triangles*** Draw $\triangle ABC$, $\triangle ABE$, $\triangle BAD$, and $\triangle BAF$.



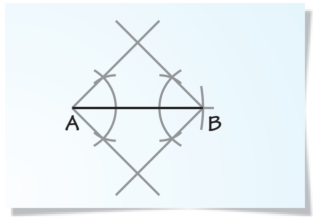
EXPLORE 2 Construct triangles from two segments and an angle (SAS)

Given two segments and one angle, like the ones at the right, use these steps to construct triangles with those side lengths and included angle.

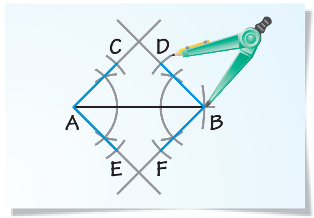


STEP 1 ***Copy a segment*** Copy one of the line segments. Label the endpoints A and B.

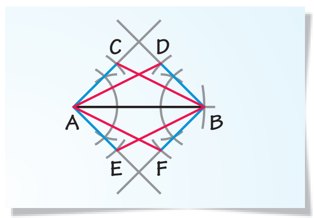
STEP 2 ***Copy angles*** Use the copy and angle construction to make two copies of the angle at point A, one above and one below the segment. Then do the same thing at point B.



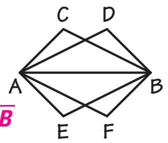
STEP 3 ***Copy segments*** Using the other given side length as radius, draw arcs with centers A and B. Label where the arcs intersect the lines from Step 2 as points C, D, E, and F.



STEP 4 ***Draw triangles*** Draw $\triangle ABC$, $\triangle ABE$, $\triangle BAD$, and $\triangle BAF$.



DRAW CONCLUSIONS Use your observations to complete these exercises



1. Refer to your work in Explore 1 and 2. Describe a rigid motion you can use to show that $\triangle ABC \cong \triangle ABE$ and $\triangle BAD \cong \triangle BAF$. **reflection in the line that contains $\overline{AB}$**
2. Refer to your work in Explore 1 and 2. Describe a rigid motion you can use to show that $\triangle ABC \cong \triangle BAD$ and $\triangle ABE \cong \triangle BAF$. **reflection in the perpendicular bisector of $\overline{AB}$**
3. Is there a rigid motion you can use to show that $\triangle ABC \cong \triangle BAF$ and $\triangle ABD \cong \triangle BAE$? If so, describe it. **yes; rotation of 180° around the midpoint of $\overline{AB}$**
4. What can you conclude about the 4 triangles in each Explore? **See margin.**
5. Follow steps like those in Explore 1 and 2 to construct triangles given two angles and an included side (ASA). Can you use rigid motions to show these triangles are all congruent? *Explain.* **See margin.**

Tips for Success

Before beginning Explore 2, make sure students are familiar with the construction for copying an angle. You may wish to review the method, which involves creating two arcs with a compass, each with a different center.

Key Discovery

Triangle criteria like three side lengths (SSS) and two side lengths and the included angle (SAS) are sufficient to generate unique triangles. Any triangles created using those criteria can be shown to be congruent to each other by finding rigid motions that map one triangle onto another.

3 ASSESS AND RETEACH

1. You have shown that a triangle built from three segments or from two segments and an included angle is unique. Explain how this is related to congruent triangles.

   Sample answer: **For any three given segment lengths, all triangles built using those lengths will be congruent. Likewise, all triangles built using two given side lengths and the angle between them will be congruent.**

4. **Rigid motions can be used to transform the triangles onto each other, so the triangles are all congruent.**

5. **Yes; the same rigid motions of reflection can be used to show that the triangles are all congruent.**

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

Suppose that $\triangle XYZ \cong \triangle RST$. Complete each statement.

1. $\overline{XY} \cong$? **$\overline{RS}$**

2. $\angle Z \cong$? **$\angle T$**

3. $m\angle S = m\angle$? **Y**

4. If $\angle A \cong \angle B$, $m\angle A = (2x + 40)°$, and $m\angle B = (3x - 10)°$, find x. **50**

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days

Average: 2 days

Advanced: 2 days

Block: 1 block

- See *Teaching Guide/Lesson Plan.*

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 2

How can you use congruent triangles to prove angles or sides congruent? **Tell students they will learn how to answer this question by using corresponding parts of congruent triangles.**

4.7 Use Congruent Triangles

Before You used corresponding parts to prove triangles congruent.

Now You will use congruent triangles to prove corresponding parts congruent.

Why? So you can find the distance across a half pipe, as in Ex. 30.

Key Vocabulary
- **corresponding parts**

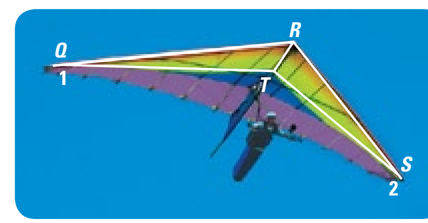
By definition, congruent triangles have congruent corresponding parts. So, if you can prove that two triangles are congruent, you know that their corresponding parts must be congruent as well.

EXAMPLE 1 Use congruent triangles

Explain how you can use the given information to prove that the hanglider parts are congruent.

GIVEN ▶ $\angle 1 \cong \angle 2$, $\angle RTQ \cong \angle RTS$

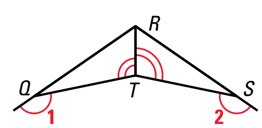
PROVE ▶ $\overline{QT} \cong \overline{ST}$



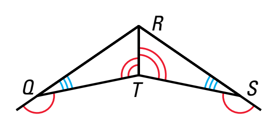
Solution

If you can show that $\triangle QRT \cong \triangle SRT$, you will know that $\overline{QT} \cong \overline{ST}$. First, copy the diagram and mark the given information. Then add the information that you can deduce. In this case, $\angle RQT$ and $\angle RST$ are supplementary to congruent angles, so $\angle RQT \cong \angle RST$. Also, $\overline{RT} \cong \overline{RT}$.

Mark given information.



Add deduced information.



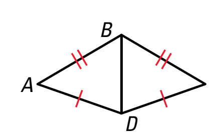
Two angle pairs and a non-included side are congruent, so by the AAS Congruence Theorem, $\triangle QRT \cong \triangle SRT$. Because corresponding parts of congruent triangles are congruent, $\overline{QT} \cong \overline{ST}$.

Animated Geometry at my.hrw.com

✓ GUIDED PRACTICE for Example 1

1. *Explain* how you can prove that $\angle A \cong \angle C$.

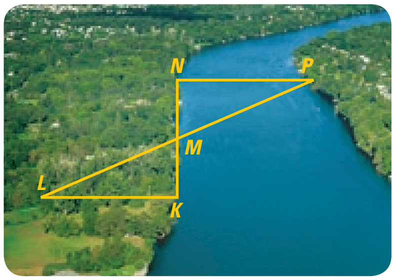
Since $\overline{BD} \cong \overline{BD}$ by the Reflexive Property, the triangles are congruent by SSS. So, $\angle A \cong \angle C$ because they are corresponding parts of congruent triangles.



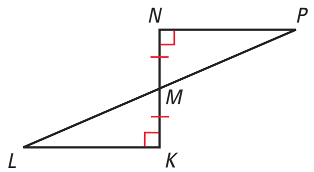
EXAMPLE 2 Use congruent triangles for measurement

INDIRECT MEASUREMENT
When you cannot easily measure a length directly, you can make conclusions about the length *indirectly,* usually by calculations based on known lengths.

SURVEYING Use the following method to find the distance across a river, from point N to point P.



- Place a stake at K on the near side so that $\overline{NK} \perp \overline{NP}$.
- Find M, the midpoint of $\overline{NK}$.
- Locate the point L so that $\overline{NK} \perp \overline{KL}$ and L, P, and M are collinear.
- Explain how this plan allows you to find the distance.



Solution

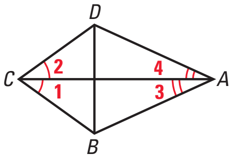
Because $\overline{NK} \perp \overline{NP}$ and $\overline{NK} \perp \overline{KL}$, $\angle N$ and $\angle K$ are congruent right angles. Because M is the midpoint of $\overline{NK}$, $\overline{NM} \cong \overline{KM}$. The vertical angles $\angle KML$ and $\angle NMP$ are congruent. So, $\triangle MLK \cong \triangle MPN$ by the ASA Congruence Postulate. Then, because corresponding parts of congruent triangles are congruent, $\overline{KL} \cong \overline{NP}$. So, you can find the distance NP across the river by measuring $\overline{KL}$.

EXAMPLE 3 Plan a proof involving pairs of triangles

Use the given information to write a plan for proof.

GIVEN ▶ $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

PROVE ▶ $\triangle BCE \cong \triangle DCE$



Solution

In $\triangle BCE$ and $\triangle DCE$, you know $\angle 1 \cong \angle 2$ and $\overline{CE} \cong \overline{CE}$. If you can show that $\overline{CB} \cong \overline{CD}$, you can use the SAS Congruence Postulate.

To prove that $\overline{CB} \cong \overline{CD}$, you can first prove that $\triangle CBA \cong \triangle CDA$. You are given $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. $\overline{CA} \cong \overline{CA}$ by the Reflexive Property. You can use the ASA Congruence Postulate to prove that $\triangle CBA \cong \triangle CDA$.

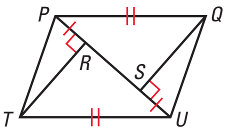
▶ **Plan for Proof** Use the ASA Congruence Postulate to prove that $\triangle CBA \cong \triangle CDA$. Then state that $\overline{CB} \cong \overline{CD}$. Use the SAS Congruence Postulate to prove that $\triangle BCE \cong \triangle DCE$.

Animated Geometry at my.hrw.com

2. No; since M is the midpoint of $\overline{NK}$, $\overline{NM} \cong \overline{MK}$. No matter how far apart the stakes at K and M are placed, the triangles will be congruent by ASA.

3. Since you already know that $\overline{TU} \cong \overline{QP}$ and $\overline{UP} \cong \overline{PU}$ you need only show $\overline{PT} \cong \overline{UQ}$ to prove the triangles are congruent by SSS. This can be done by showing right triangles QSP and TRU are congruent by HL leading to right triangles USQ and PRT being congruent by HL which gives you $\overline{PT} \cong \overline{UQ}$.

✓ GUIDED PRACTICE for Examples 2 and 3



2. In Example 2, does it matter how far from point N you place a stake at point K? *Explain.* **See margin.**
3. Using the information in the diagram at the right, write a plan to prove that $\triangle PTU \cong \triangle UQP$. **See margin.**

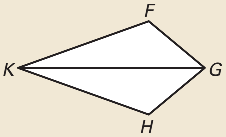
Motivating the Lesson

Tell students that congruent triangles can be used to find distances that are difficult to measure directly. Tell them that in this lesson they will see some examples of how to do this.

3 TEACH

Extra Example 1

Use the given information to prove the parts of the kite are congruent.
Given: $\overline{GK}$ bisects $\angle FGH$ and $\angle FKH$.
Prove: $\overline{FK} \cong \overline{HK}$



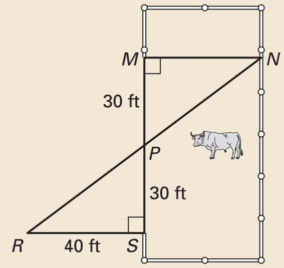
$\angle FGK \cong \angle HGK$ and $\angle FKG \cong \angle HKG$ by the def. of $\angle$ bisector. $\overline{GK} \cong \overline{GK}$ by the Refl. Prop. of $\cong$ Segs. Therefore $\triangle FGK \cong \triangle HGK$ by the ASA Cong. Post. So $\overline{FK} \cong \overline{HK}$ since Corr. Parts of $\cong$ $\triangle$s are $\cong$.

Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for **Example 1**. This activity is also part of **Power Presentations**.

Extra Example 2

If P is the midpoint of $\overline{MS}$, how wide is the bull's pasture? **40 ft**



Differentiated Instruction

Kinesthetic Learners Take students outside to construct and solve a problem similar to **Example 2**. Find a distance that may be difficult to measure directly with a measuring tape and use congruent triangles to find the distance. Direct students to draw a diagram of the problem, make measurements, write distance measurements on the diagram, and use these measurements and the appropriate postulate to find the distance.

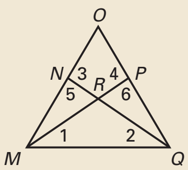
See also the *Differentiated Instruction Resources* for more strategies.

Extra Example 3

Write a plan for a proof.

Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

Prove: $\triangle MNR \cong \triangle QPR$



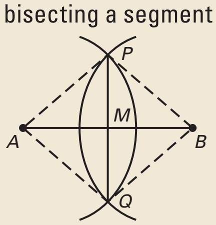
By the Cong. Supplements Thm. $\angle 5 \cong \angle 6$. By the Refl. Prop. of $\cong$ Segs. $\overline{MQ} \cong \overline{MQ}$. By the AAS Cong. Thm. $\triangle MNQ \cong \triangle QPM$. Since corr. parts of $\cong$ ⚠ are $\cong$, $\overline{MN} \cong \overline{PQ}$. By the Vert. ⚠ Thm. $\angle MRN \cong \angle QRP$. By the AAS Cong. Thm. $\triangle MNR \cong \triangle QPR$.

Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for **Example 3**. This activity is also part of **Power Presentations**.

Extra Example 4

Prove that the construction for bisecting a segment is valid.



Since the compass setting is the same, $\overline{AP} \cong \overline{BP}$ and $\overline{AQ} \cong \overline{BQ}$. By the Refl. Prop. of $\cong$, $\overline{PQ} \cong \overline{PQ}$. So $\triangle APQ \cong \triangle BPQ$ by SSS. Corresponding parts of $\cong$ ⚠ are $\cong$, so $\angle APM \cong \angle BPM$. Since $\overline{PM} \cong \overline{PM}$ (Refl. Prop. of $\cong$), $\triangle APM \cong \triangle BPM$ by SAS. Hence the corr. sides $\overline{AM}$ and $\overline{BM}$ are $\cong$ and $\overline{PQ}$ bisects $\overline{AB}$ by the def. of bisector.

Closing the Lesson

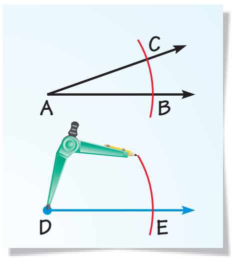
Have students summarize the major points of the lesson and answer the Essential Question: How can you use congruent triangles to prove angles or sides congruent?

- **If triangles are congruent, their corresponding parts are congruent.**

Use the fact that Corr. Parts of $\cong$ ⚠ are $\cong$.

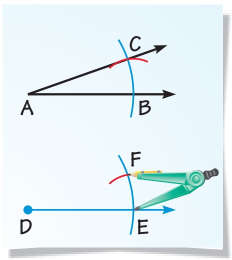
PROVING CONSTRUCTIONS You have learned how to use a compass and a straightedge to copy an angle. The construction is shown below. You can use congruent triangles to prove that this construction is valid.

STEP 1



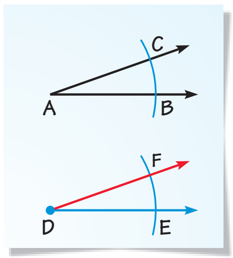
To copy $\angle A$, draw a segment with initial point D. Draw an arc with center A. Using the same radius, draw an arc with center D. Label points B, C, and E.

STEP 2



Draw an arc with radius BC and center E. Label the intersection F.

STEP 3



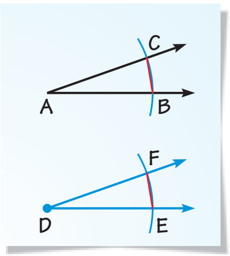
Draw $\overrightarrow{DF}$. In Example 4, you will prove that $\angle D \cong \angle A$.

EXAMPLE 4 Prove a construction

Write a proof to verify that the construction for copying an angle is valid.

Solution

Add $\overline{BC}$ and $\overline{EF}$ to the diagram. In the construction, $\overline{AB}$, $\overline{DE}$, $\overline{AC}$, and $\overline{DF}$ are all determined by the same compass setting, as are $\overline{BC}$ and $\overline{EF}$. So, you can assume the following as given statements.



GIVEN ▶ $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$

PROVE ▶ $\angle D \cong \angle A$

Plan for Proof Show that $\triangle CAB \cong \triangle FDE$, so you can conclude that the corresponding parts $\angle A$ and $\angle D$ are congruent.

Plan in Action

STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$	1. Given
2. $\triangle FDE \cong \triangle CAB$	2. SSS Congruence Postulate
3. $\angle D \cong \angle A$	3. Corresp. parts of $\cong$ ⚠ are $\cong$.

✓ **GUIDED PRACTICE** for Example 4

4. Find the construction of an angle bisector. What segments can you assume are congruent? **$\overline{AC}$ and $\overline{AB}$**

4.7 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 19, 23, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 14, and 31

SKILL PRACTICE

A

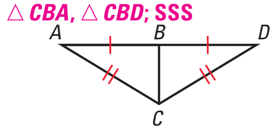
1. **VOCABULARY** Copy and complete: Corresponding parts of congruent triangles are _?_. **congruent**

2. ★ **WRITING** *Explain* why you might choose to use congruent triangles to measure the distance across a river. Give another example where it may be easier to measure with congruent triangles rather than directly.
***Sample answer:* You are unable to cross the river; measuring the distance across a lake.**

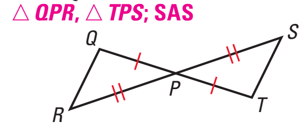
EXAMPLES 1 and 2 for Exs. 3–11

CONGRUENT TRIANGLES **Tell which triangles you can show are congruent in order to prove the statement. What postulate or theorem would you use?**

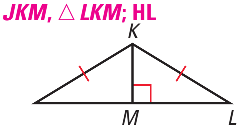
3. $\angle A \cong \angle D$
$\triangle CBA$, $\triangle CBD$; SSS



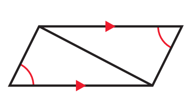
4. $\angle Q \cong \angle T$
$\triangle QPR$, $\triangle TPS$; SAS



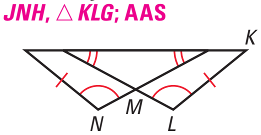
5. $\overline{JM} \cong \overline{LM}$
$\triangle JKM$, $\triangle LKM$; HL



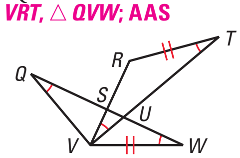
6. $\overline{AC} \cong \overline{BD}$
$\triangle CAD$, $\triangle BDA$; AAS



7. $\overline{GK} \cong \overline{HJ}$
$\triangle JNH$, $\triangle KLG$; AAS

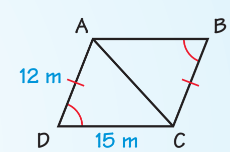


8. $\overline{QW} \cong \overline{TV}$
$\triangle VRT$, $\triangle QVW$; AAS



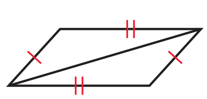
9. **ERROR ANALYSIS** *Describe* the error in the statement.
The angle is not the included angle; the triangles cannot be said to be congruent.

$\triangle ABC \cong \triangle CDA$ by SAS.
So, $AB = 15$ meters.

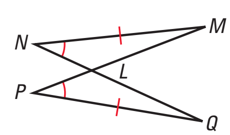


PLANNING FOR PROOF **Use the diagram to write a plan for proof.**

10. **PROVE** ▶ $\angle S \cong \angle U$



11. **PROVE** ▶ $\overline{LM} \cong \overline{LQ}$



10. Show $\triangle VST \cong \triangle TUV$ by SSS since $\overline{VT} \cong \overline{TV}$ by the Reflexive Property of Congruence. Then $\angle S \cong \angle U$ because corresponding parts of congruent triangles are congruent.

11. Show $\triangle NML \cong \triangle PQL$ by AAS since $\angle NLM \cong \angle PLQ$ by the Vertical Angles Congruence Theorem. Then $\overline{LM} \cong \overline{LQ}$ because corresponding parts of congruent triangles are congruent.

B

12. **PENTAGONS** *Explain* why segments connecting any pair of corresponding vertices of congruent pentagons are congruent. Make a sketch to support your answer. **Corresponding diagonals are corresponding sides of two congruent triangles; see margin for art.**

13. **xy ALGEBRA** Given that $\triangle ABC \cong \triangle DEF$, $m\angle A = 70°$, $m\angle B = 60°$, $m\angle C = 50°$, $m\angle D = (3x + 10)°$, $m\angle E = \left(\frac{y}{3} + 20\right)°$, and $m\angle F = (z^2 + 14)°$, find the values of x, y, and z. **20, 120, ±6**

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–11, 28
Day 2:
Exs. 12–17, 29–33

Average:
Day 1:
Exs. 1–11, 28
Day 2:
Exs. 12–14, 18–24, 29–36

Advanced:
Day 1:
Exs. 1, 2, 4–11, 27*, 28
Day 2:
Exs. 12–14, 19–26, 31–40*

Block:
Exs. 1–14, 18–24, 28–36

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 4, 10, 16, 28, 32
Average: 6, 10, 18, 28, 32
Advanced: 8, 11, 20, 28, 32

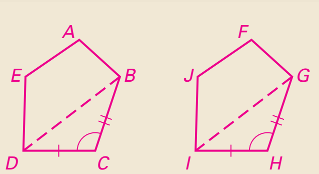
Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

12. *Sample:*



Avoiding Common Errors

Exercise 6 Students may say that $\triangle ACD \cong \triangle DBC$ because $\angle CAD \cong \angle BDA$ are congruent alternate interior angles. These angles are congruent, but point out that we do not know this until after we have proved that the triangles are congruent.

Study Strategy

Exercise 13 Tell students to draw and label a diagram before they try to write and solve the equations.

Reading Strategy

Exercise 15 Students should be careful reading the labels on the figure. $\angle 1$ is $\angle KFJ$ not the entire right angle. Likewise $\angle 2$ is $\angle HGJ$.

15. Show $\triangle KFG \cong \triangle HGF$ by AAS, which gives you $\overline{HG} \cong \overline{KF}$. This along with $\angle FJK \cong \angle GJH$ by vertical angles gives you $\triangle FJK \cong \triangle GJH$ by AAS, therefore $\angle 1 \cong \angle 2$.

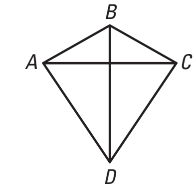
16. $\triangle AEB \cong \triangle DEC$ by AAS which makes $\angle ABE \cong \angle DCE$. Then by the Congruent Supplements Theorem, $\angle 1 \cong \angle 2$.

17. Show $\triangle STR \cong \triangle QTP$ by ASA using the givens and vertical angles STR and QTP. Since $\overline{PT} \cong \overline{RT}$ and using vertical angles PTS and RTQ, $\triangle PTS \cong \triangle RTQ$ by SAS, which gives you $\angle 1 \cong \angle 2$.

18. Show $\triangle ABE \cong \triangle CBE$ by ASA, which gives you $\overline{AE} \cong \overline{CE}$. Use the Angle Addition Postulate and congruent angles to show $\angle FAE \cong \angle DCE$. Then $\triangle AEF \cong \triangle CED$ by SAS, and $\angle 1 \cong \angle 2$.

19. Show $\triangle KNP \cong \triangle MNP$ by SSS. Now $\angle KPL \cong \angle MPL$ and $\overline{PL} \cong \overline{PL}$ leads to $\triangle LKP \cong \triangle LMP$ by SAS, which gives you $\angle 1 \cong \angle 2$.

14. ★ **MULTIPLE CHOICE** Which set of given information does *not* allow you to conclude that $\overline{AD} \cong \overline{CD}$? **B**



Ⓐ $\overline{AE} \cong \overline{CE}$, $m\angle BEA = 90°$

Ⓑ $\overline{BA} \cong \overline{BC}$, $\angle BDC \cong \angle BDA$

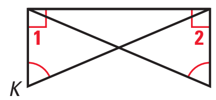
Ⓒ $\overline{AB} \cong \overline{CB}$, $\angle ABE \cong \angle CBE$

Ⓓ $\overline{AE} \cong \overline{CE}$, $\overline{AB} \cong \overline{CB}$

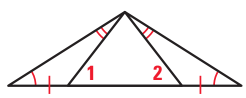
EXAMPLE 3 for Exs. 15–20

PLANNING FOR PROOF Use the information given in the diagram to write a plan for proving that $\angle 1 \cong \angle 2$. **15–20. See margin.**

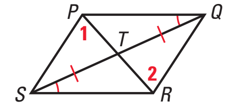
15.



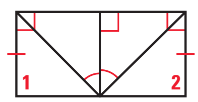
16.



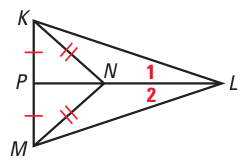
17.



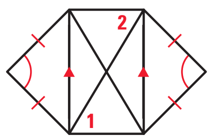
18.



19.



20.



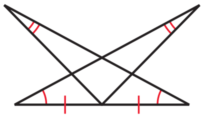
USING COORDINATES Use the vertices of $\triangle ABC$ and $\triangle DEF$ to show that $\angle A \cong \angle D$. *Explain* your reasoning.

21. $A(3, 7)$, $B(6, 11)$, $C(11, 13)$, $D(2, -4)$, $E(5, -8)$, $F(10, -10)$ **The triangles are congruent by SSS.**

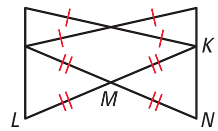
22. $A(3, 8)$, $B(3, 2)$, $C(11, 2)$, $D(-1, 5)$, $E(5, 5)$, $F(5, 13)$ **The triangles are congruent by SSS.**

PROOF Use the information given in the diagram to write a proof. **23–26. See margin.**

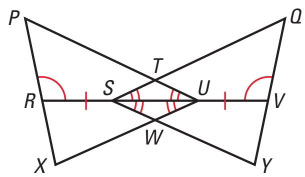
23. **PROVE** ▶ $\angle VYX \cong \angle WYZ$



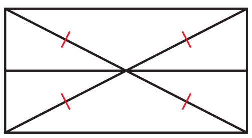
24. **PROVE** ▶ $\overline{FL} \cong \overline{HN}$



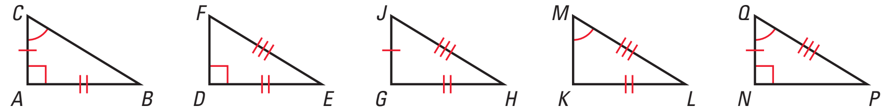
C **25.** **PROVE** ▶ $\triangle PUX \cong \triangle QSY$



26. **PROVE** ▶ $\overline{AC} \cong \overline{GE}$



27. **CHALLENGE** Which of the triangles below are congruent? **$\triangle ABC$, $\triangle NPQ$, $\triangle DEF$, and $\triangle GHJ$**



◯ = See **WORKED-OUT SOLUTIONS** in Student Resources ★ = **STANDARDIZED TEST PRACTICE**

20. Since $\triangle TVY \cong \triangle UXZ$ by SAS you have $\overline{YT} \cong \overline{ZU}$. Since $\overline{YT} \parallel \overline{ZU}$, you have $\angle YTW \cong \angle UZW$ and $\angle TYW \cong \angle ZUW$ by the Alternate Interior Angles Theorem, making $\triangle TYW \cong \triangle ZUW$ by ASA. Using corresponding parts and vertical angles, you have $\triangle TWU \cong \triangle ZWY$ by SAS, making $\angle 1 \cong \angle 2$.

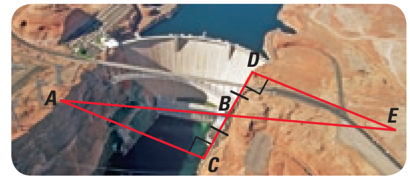
23–26. See Additional Answers.

PROBLEM SOLVING

EXAMPLE 2 A for Ex. 28

28. Because $\overline{CD} \perp \overline{DE}$ and $\overline{CD} \perp \overline{AC}$, $\angle D$ and $\angle C$ are congruent right angles. The vertical angles, $\angle DBE$ and $\angle CBA$, are congruent. So, $\triangle DBE \cong \triangle CBA$ by ASA. Then because corresponding parts of congruent triangles are congruent, $\overline{AC} \cong \overline{DE}$. So, you can find the distance AC across the canyon by measuring DE.

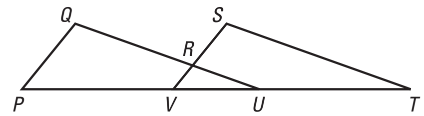
28. CANYON *Explain* how you can find the distance across the canyon.



29. PROOF Use the given information and the diagram to write a two-column proof.

GIVEN ▶ $\overline{PQ} \parallel \overline{VS}$, $\overline{QU} \parallel \overline{ST}$, $\overline{PQ} \cong \overline{VS}$

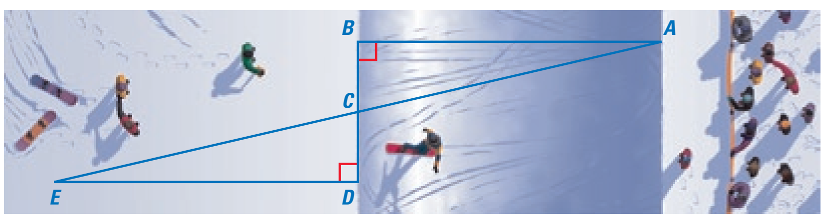
PROVE ▶ $\angle Q \cong \angle S$ **See margin.**



29. Statements (Reasons)

1. $\overline{PQ} \parallel \overline{VS}$, $\overline{QU} \parallel \overline{ST}$, $\overline{PQ} \cong \overline{VS}$ (Given)
2. $\angle QPU \cong \angle SVT$, $\angle QUP \cong \angle STV$ (Corresponding Angles Postulate)
3. $\triangle PQU \cong \triangle VST$ (AAS)
4. $\angle Q \cong \angle S$ (Corr. parts of $\cong$ ▵ are $\cong$.)

30. SNOWBOARDING In the diagram of the half pipe below, C is the midpoint of $\overline{BD}$. If $EC \approx 11.5$ m, and $CD \approx 2.5$ m, find the approximate distance across the half pipe. *Explain* your reasoning.



11.2 m. *Sample answer:* $\triangle ABC \cong \triangle EDC$ thus $\overline{ED} \cong \overline{AB}$. Since $ED \approx 11.2$ then $AB = 11.2$.

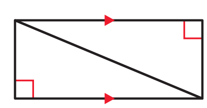
31. ★ **MULTIPLE CHOICE** Using the information in the diagram, you can prove that $\overline{WY} \cong \overline{ZX}$. Which reason would *not* appear in the proof? **A**

Ⓐ SAS Congruence Postulate

Ⓑ AAS Congruence Theorem

Ⓒ Alternate Interior Angles Theorem

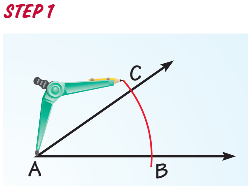
Ⓓ Right Angles Congruence Theorem



EXAMPLE 4 B for Ex. 32

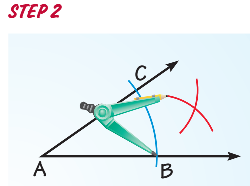
32. PROVING A CONSTRUCTION The diagrams below show the construction that used to bisect $\angle A$. By construction, you can assume that $\overline{AB} \cong \overline{AC}$ and $\overline{BG} \cong \overline{CG}$. Write a proof to verify that $\overrightarrow{AG}$ bisects $\angle A$. **See margin.**

STEP 1



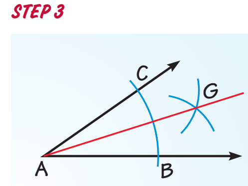
First draw an arc with center A. Label the points where the arc intersects the sides of the angle points B and C.

STEP 2



Draw an arc with center C. Using the same radius, draw an arc with center B. Label the intersection point G.

STEP 3



Draw $\overrightarrow{AG}$. It follows that $\angle BAG \cong \angle CAG$.

32. Statements (Reasons)

1. $\overline{AB} \cong \overline{AC}$, $\overline{BG} \cong \overline{CG}$ (Given)
2. $\overline{AG} \cong \overline{AG}$ (Reflexive Property of Segment Congruence)
3. $\triangle ACG \cong \triangle ABG$ (SSS)
4. $\angle CAG \cong \angle BAG$ (Corr. parts of $\cong$ ▵ are $\cong$.)
5. $\overrightarrow{AG}$ bisects $\angle A$. (Definition of angle bisector)

Study Strategy

Exercises 38–39 Draw the overlapping triangles apart from each other. Mark the parts congruent for the first pair of triangles you are planning to prove congruent in a different color than the parts you are going to use for the second pair of triangles.

37. Statements (Reasons)

1. $\overline{MN} \cong \overline{KN}$, $\angle PMN \cong \angle NKL$ (Given)
2. $\angle MNP \cong \angle KNL$ (Vertical Angles Congruence Theorem)
3. $\triangle PMN \cong \triangle LKN$ (ASA)
4. $\overline{MP} \cong \overline{KL}$, $\angle MPJ \cong \angle KLQ$ (Corr. parts of $\cong$ $\triangle$s are $\cong$.)
5. $\overline{MJ} \perp \overline{PN}$, $\overline{KQ} \perp \overline{LN}$ (Given in diagram)
6. $\angle KQL$ and $\angle MJP$ are right angles. (Perpendicular lines intersect to form four right angles.)
7. $\angle KQL \cong \angle MJP$ (Right Angles Congruence Theorem)
8. $\triangle MJP \cong \triangle KQL$ (AAS)
9. $\angle 1 \cong \angle 2$ (Corr. parts of $\cong$ $\triangle$s are $\cong$.)

38. Statements (Reasons)

1. $\overline{TS} \cong \overline{TV}$, $\overline{SR} \cong \overline{VW}$ (Given)
2. $TS = TV$, $SR = VW$ (Definition of congruent segments)
3. $TS + SR = TR$, $TV + VW = TW$ (Segment Addition Postulate)
4. $TV + SR = TR$, $TV + SR = TW$ (Substitution Property of Equality)
5. $TR = TW$ (Transitive Property of Equality)
6. $\overline{TR} \cong \overline{TW}$ (Definition of congruent segments)
7. $\angle RTV \cong \angle WTS$ (Reflexive Property of Congruence)
8. $\triangle RTV \cong \triangle WTS$ (SAS)
9. $\overline{RV} \cong \overline{WS}$ (Corr. parts of $\cong$ $\triangle$s are $\cong$.)
10. $\overline{SV} \cong \overline{VS}$ (Reflexive Property of Congruence)
11. $\triangle RSV \cong \triangle WVS$ (SSS)
12. $\angle RSV \cong \angle WVS$ (Corr. parts of $\cong$ $\triangle$s are $\cong$.)
13. $\angle RSV$ and $\angle 1$ are supplementary; $\angle WVS$ and $\angle 2$ are supplementary. (Linear Pair Postulate)
14. $\angle 1 \cong \angle 2$ (Congruent Supplements Theorem)

33. No; the given angle is not an included angle.

34. Yes; $\overline{AE} \cong \overline{CE}$ by Corr. parts of $\cong$ $\triangle$s are $\cong$, $\angle CEB \cong \angle AEB$ by the Right Angle Congruence Theorem and $\overline{BE} \cong \overline{BE}$ so $\triangle BAE \cong \triangle BCE$. By Corr. parts of $\cong$ $\triangle$s are $\cong$, $\overline{AB} \cong \overline{BC}$.

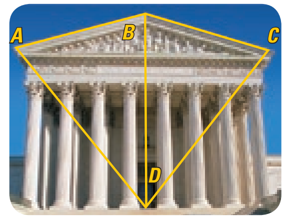
35. Yes; $\angle BDA \cong \angle BDC$, $\overline{AD} \cong \overline{CD}$ and $\overline{BD} \cong \overline{BD}$. By SAS, $\triangle ABD \cong \triangle CBD$. By Corr. parts of $\cong$ $\triangle$s are $\cong$, $\overline{AB} \cong \overline{BC}$.

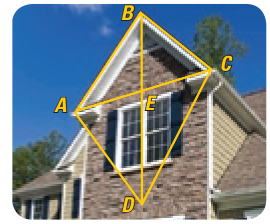
ARCHITECTURE **Can you use the given information to determine that $\overline{AB} \cong \overline{BC}$? *Justify* your answer.** 33–35. See margin.

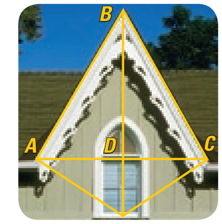
33. $\angle ABD \cong \angle CBD$, $AD = CD$

34. $\overline{AC} \perp \overline{BD}$, $\triangle ADE \cong \triangle CDE$

35. $\overline{BD}$ bisects $\overline{AC}$, $\overline{AD} \perp \overline{BD}$

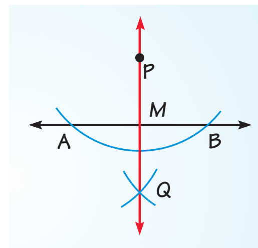






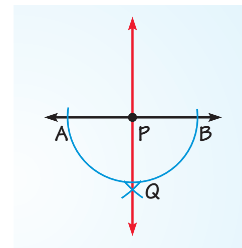
36. CONSTRUCTIONS In parts (a) and (b), write a proof to justify the following constructions. See margin.

a. Line perpendicular to a line through a point *not* on the line.



Plan for Proof Show $\triangle APQ \cong \triangle BPQ$ by SSS. Then show $\triangle APM \cong \triangle BPM$ using SAS. Use corresponding parts of congruent triangles to show that $\angle AMP$ and $\angle BMP$ are right angles.

b. Line perpendicular to a line through a point *on* the line.

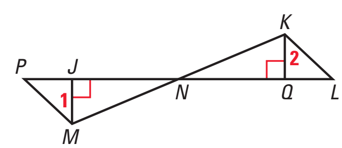


Plan for Proof Show $\triangle APQ \cong \triangle BPQ$ by SSS. Use corresponding parts of congruent triangles to show that $\angle QPA$ and $\angle QPB$ are right angles.

PROOF **Use the given information and the diagram to write a proof.** 37–39. See margin.

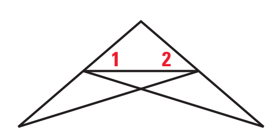
37. **GIVEN** ▶ $\overline{MN} \cong \overline{KN}$, $\angle PMN \cong \angle NKL$

PROVE ▶ $\angle 1 \cong \angle 2$



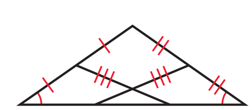
C **38.** **GIVEN** ▶ $\overline{TS} \cong \overline{TV}$, $\overline{SR} \cong \overline{VW}$

PROVE ▶ $\angle 1 \cong \angle 2$



39. **GIVEN** ▶ $\overline{BA} \cong \overline{BC}$, D and E are midpoints, $\angle A \cong \angle C$, $\overline{DF} \cong \overline{EF}$

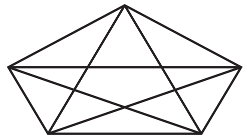
PROVE ▶ $\overline{FG} \cong \overline{FH}$



○ = See **WORKED-OUT SOLUTIONS** in Student Resources ★ = **STANDARDIZED TEST PRACTICE**

36, 39. See Additional Answers.

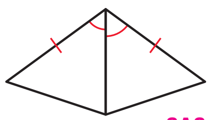
40. **CHALLENGE** In the diagram of pentagon $ABCDE$, $\overline{AB} \parallel \overline{EC}$, $\overline{AC} \parallel \overline{ED}$, $\overline{AB} \cong \overline{ED}$, and $\overline{AC} \cong \overline{EC}$. Write a proof that shows $\overline{AD} \cong \overline{EB}$. **See margin.**



QUIZ

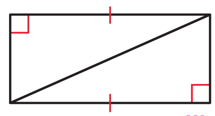
Decide which method, SAS, ASA, AAS, or HL, can be used to prove that the triangles are congruent.

1.



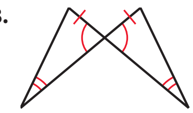
SAS

2.



HL

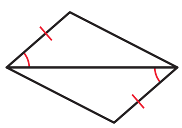
3.



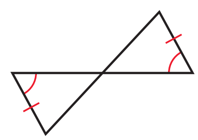
AAS

Use the given information to write a proof. **4–6. See margin.**

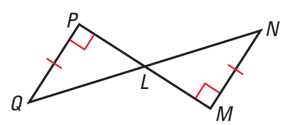
4. **GIVEN** ▶ $\angle BAC \cong \angle DCA$, $\overline{AB} \cong \overline{CD}$
PROVE ▶ $\triangle ABC \cong \triangle CDA$



5. **GIVEN** ▶ $\angle W \cong \angle Z$, $\overline{VW} \cong \overline{YZ}$
PROVE ▶ $\triangle VWX \cong \triangle YZX$



6. Write a plan for a proof.
GIVEN ▶ $\overline{PQ} \cong \overline{MN}$, $m\angle P = m\angle M = 90°$
PROVE ▶ $\overline{QL} \cong \overline{NL}$

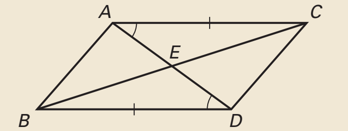


5 ASSESS AND RETEACH

Daily Homework Quiz

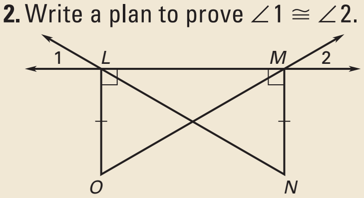
Also available online

1. Tell which triangles you can show are congruent in order to prove $AE = DE$. What postulate or theorem would you use?



$\triangle AEC \cong \triangle DEB$ by the AAS Cong. Thm. or by the ASA Cong. Post.

2. Write a plan to prove $\angle 1 \cong \angle 2$.



Show $\overline{LM} \cong \overline{LM}$ by the Refl. Prop. of $\cong$ Segs. Hence $\triangle OLM \cong \triangle NML$ by the SAS Cong. Post. This gives $\angle NLM \cong \angle OML$, since Corr. Parts of $\cong$ △s are $\cong$. So $\angle 1 \cong \angle 2$ by the Vert. △s Thm. and properties of $\cong$ △s.

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

Quiz

An easily-readable reduced copy of the quiz from the Assessment Book can be found at the beginning of this chapter.

40, Quiz 4–6. See Additional Answers.

1 PLAN AND PREPARE

Warm-Up Exercises

Also available online

Classify each triangle by its sides.

1. 2 cm, 2 cm, 2 cm **equilateral**
2. 7 ft, 11 ft, 7 ft **isosceles**
3. 9 m, 8 m, 10 m **scalene**
4. In $\triangle ABC$, if $m\angle A = 70°$ and $m\angle B = 50°$, what is $m\angle C$? **60°**
5. In $\triangle DEF$, if $m\angle D = m\angle E$ and $m\angle F = 26°$, what are the measures of $\angle D$ and $\angle E$? **77°, 77°**

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days
Average: 2 days
Advanced: 2 days
Block: 1 block

- See *Teaching Guide/Lesson Plan*.

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 2

How are the sides and angles of a triangle related if there are two or more congruent sides or angles? **Tell students they will learn how to answer this question by learning the Base Angles Theorem and its converse.**

4.8 Use Isosceles and Equilateral Triangles

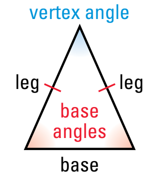
Before You learned about isosceles and equilateral triangles.

Now You will use theorems about isosceles and equilateral triangles.

Why? So you can solve a problem about architecture, as in Ex. 40.

Key Vocabulary
- **legs**
- **vertex angle**
- **base**
- **base angles**

A triangle is isosceles if it has at least two congruent sides. When an isosceles triangle has exactly two congruent sides, these two sides are the **legs**. The angle formed by the legs is the **vertex angle**. The third side is the **base** of the isosceles triangle. The two angles adjacent to the base are called **base angles**.

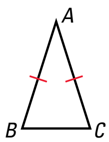


THEOREMS — *For Your Notebook*

THEOREM 4.7 Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

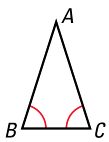
If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.



THEOREM 4.8 Converse of Base Angles Theorem

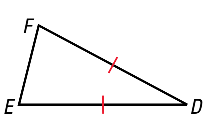
If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.



EXAMPLE 1 Apply the Base Angles Theorem

In $\triangle DEF$, $\overline{DE} \cong \overline{DF}$. Name two congruent angles.



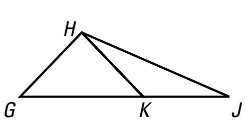
Solution

▸ $\overline{DE} \cong \overline{DF}$, so by the Base Angles Theorem, $\angle E \cong \angle F$.

GUIDED PRACTICE for Example 1

Copy and complete the statement.

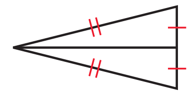
1. If $\overline{HG} \cong \overline{HK}$, then $\angle$ _?_ $\cong \angle$ _?_. ***HGK, HKG***
2. If $\angle KHJ \cong \angle KJH$, then _?_ $\cong$ _?_. ***$\overline{KH}$, $\overline{KJ}$***



PROOF Base Angles Theorem

GIVEN ▶ $\overline{JK} \cong \overline{JL}$

PROVE ▶ $\angle K \cong \angle L$



Plan for Proof
a. Draw $\overline{JM}$ so that it bisects $\overline{KL}$.
b. Use SSS to show that $\triangle JMK \cong \triangle JML$.
c. Use properties of congruent triangles to show that $\angle K \cong \angle L$.

Plan in Action

	STATEMENTS	REASONS
	1. M is the midpoint of $\overline{KL}$.	1. Definition of midpoint
a.	2. Draw $\overline{JM}$.	2. Two points determine a line.
	3. $\overline{MK} \cong \overline{ML}$	3. Definition of midpoint
	4. $\overline{JK} \cong \overline{JL}$	4. Given
	5. $\overline{JM} \cong \overline{JM}$	5. Reflexive Property of Congruence
b.	6. $\triangle JMK \cong \triangle JML$	6. SSS Congruence Postulate
c.	7. $\angle K \cong \angle L$	7. Corresp. parts of $\cong$ $\triangle$s are $\cong$.

Recall that an equilateral triangle has three congruent sides.

WRITE A BICONDITIONAL
The corollaries state that a triangle is *equilateral* if and only if it is *equiangular.*

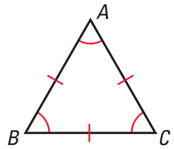
COROLLARIES *For Your Notebook*

Corollary to the Base Angles Theorem

If a triangle is equilateral, then it is equiangular.

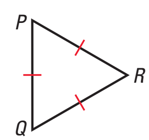
Corollary to the Converse of Base Angles Theorem

If a triangle is equiangular, then it is equilateral.



EXAMPLE 2 Find measures in a triangle

Find the measures of $\angle P$, $\angle Q$, and $\angle R$.



The diagram shows that $\triangle PQR$ is equilateral. Therefore, by the Corollary to the Base Angles Theorem, $\triangle PQR$ is equiangular. So, $m\angle P = m\angle Q = m\angle R$.

$3(m\angle P) = 180°$ **Triangle Sum Theorem**

$m\angle P = 60°$ **Divide each side by 3.**

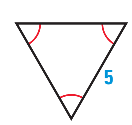
▶ The measures of $\angle P$, $\angle Q$, and $\angle R$ are all 60°.

✓ GUIDED PRACTICE for Example 2

3. Find ST in the triangle at the right. **5**

4. Is it possible for an equilateral triangle to have an angle measure other than 60°? *Explain.*

No. *Sample answer:* The Triangle Sum Theorem and the fact that the triangle is equilateral guarantees the angles measure 60° because all pairs of angles could be considered base angles of an isosceles triangle.



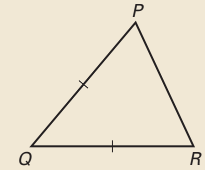
Motivating the Lesson

Isosceles and equilateral triangles appear in many figures that students will study in art and architecture. Tell students that in this lesson, they will learn properties of triangles that have two or more congruent sides or angles.

3 TEACH

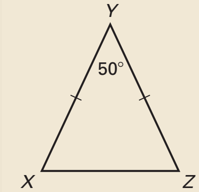
Extra Example 1

In $\triangle PQR$, $\overline{PQ} \cong \overline{QR}$. Name two congruent angles. **$\angle P \cong \angle R$**



Extra Example 2

Find the measures of $\angle X$ and $\angle Z$. **65°, 65°**



Key Question to Ask for Example 2

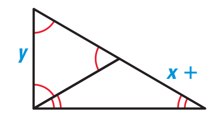
- Can an equilateral triangle have an angle of 61°? Explain. **No; each angle must have a measure of 60°.**

Vocabulary

Discuss with students whether each side of an equilateral triangle can be called a leg of the triangle. Call attention to how the legs of an isosceles triangle are defined in previous page. Have students note that the word is applied only to isosceles triangles that have exactly two congruent sides.

EXAMPLE 3 Use isosceles and equilateral triangles

xy ALGEBRA Find the values of x and y in the diagram.



Solution

STEP 1 **Find** the value of y. Because $\triangle KLN$ is equiangular, it is also equilateral and $\overline{KN} \cong \overline{KL}$. Therefore, $y = 4$.

STEP 2 **Find** the value of x. Because $\angle LNM \cong \angle LMN$, $\overline{LN} \cong \overline{LM}$ and $\triangle LMN$ is isosceles. You also know that $LN = 4$ because $\triangle KLN$ is equilateral.

$LN = LM$	**Definition of congruent segments**
$4 = x + 1$	**Substitute 4 for *LN* and *x* + 1 for *LM*.**
$3 = x$	**Subtract 1 from each side.**

AVOID ERRORS
You cannot use $\angle N$ to refer to $\angle LNM$ because three angles have N as their vertex.

EXAMPLE 4 Solve a multi-step problem

LIFEGUARD TOWER In the lifeguard tower, $\overline{PS} \cong \overline{QR}$ and $\angle QPS \cong \angle PQR$.

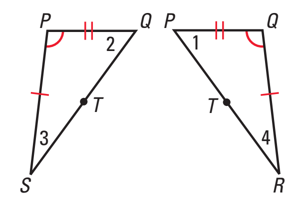
a. What congruence postulate can you use to prove that $\triangle QPS \cong \triangle PQR$?

b. Explain why $\triangle PQT$ is isosceles.

c. Show that $\triangle PTS \cong \triangle QTR$.

Solution

a. Draw and label $\triangle QPS$ and $\triangle PQR$ so that they do not overlap. You can see that $\overline{PQ} \cong \overline{QP}$, $\overline{PS} \cong \overline{QR}$, and $\angle QPS \cong \angle PQR$. So, by the SAS Congruence Postulate, $\triangle QPS \cong \triangle PQR$.



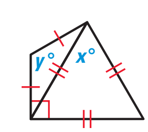
b. From part (a), you know that $\angle 1 \cong \angle 2$ because corresp. parts of $\cong$ △ are $\cong$. By the Converse of the Base Angles Theorem, $\overline{PT} \cong \overline{QT}$, and $\triangle PQT$ is isosceles.

c. You know that $\overline{PS} \cong \overline{QR}$, and $\angle 3 \cong \angle 4$ because corresp. parts of $\cong$ △ are $\cong$. Also, $\angle PTS \cong \angle QTR$ by the Vertical Angles Congruence Theorem. So, $\triangle PTS \cong \triangle QTR$ by the AAS Congruence Theorem.

AVOID ERRORS
When you redraw the triangles so that they do not overlap, be careful to copy all given information and labels correctly.

Jeff Baker/Getty Images

✓ GUIDED PRACTICE for Examples 3 and 4

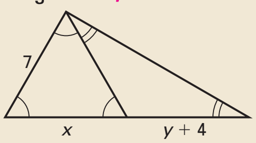


5. Find the values of x and y in the diagram. **60, 120**

6. REASONING Use parts (b) and (c) in Example 4 and the SSS Congruence Postulate to give a different proof that $\triangle QPS \cong \triangle PQR$. **See margin.**

Extra Example 3

Find the values of x and y in the diagram. **7, 3**



Key Question to Ask for Example 3

- Explain how you could find $m\angle M$. **$m\angle KLN = 60°$ because the triangle is equilateral. So $m\angle MLN = 120°$. That leaves 60° for $m\angle M + m\angle MNL$, and since $m\angle M$ and $m\angle MNL$ are equal, each must be 30°.**

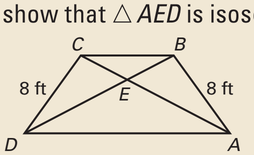
Extra Example 4

Diagonal braces $\overline{AC}$ and $\overline{BD}$ are used to reinforce a signboard that advertises fresh eggs and produce at a roadside stand. Each brace is 14 feet long.

a. What congruence postulate can you use to prove that $\triangle ABC \cong \triangle DCB$? **SSS Cong. Post.**

b. Explain why $\triangle BEC$ is isosceles. **$\angle DBC \cong \angle ACB$ since corr. parts of $\cong$ △ are $\cong$. $\overline{BE} \cong \overline{CE}$ by the Conv. of the Base Angles Thm., and this implies that $\triangle BEC$ is isosceles.**

c. What triangles would you use to show that $\triangle AED$ is isosceles?



$\triangle ABD$ and $\triangle DCA$

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How are the sides and angles of a triangle related if there are two or more congruent sides or angles?

- **Angles opposite congruent sides of a triangle are congruent and conversely.**
- **If a triangle is equilateral, then it is equiangular and conversely.**

If two sides of a triangle are congruent, then the angles opposite them are congruent. The converse is also true.

6. By the Segment Addition Postulate $QT + TS = QS$ and $PT + TR = PR$. Since $\overline{PT} \cong \overline{QT}$ from part (b) and $\overline{TS} \cong \overline{TR}$ from part (c), then $\overline{QS} \cong \overline{PR}$. $\overline{PQ} \cong \overline{PQ}$ by the Reflexive Property and it is given that $\overline{PS} \cong \overline{QR}$, therefore $\triangle QPS \cong \triangle PQR$ by the SSS Congruence Postulate.

Differentiated Instruction

Inclusion Some students may not understand where to begin to find the values of x and y in **Guided Practice Exercise 5**. Guide them by asking the following questions: "What can you say about the angles of a triangle with three congruent sides? What is the measure of each angle in an equilateral triangle? How can you find the measures of the two angles that make up the 90° angle using information about the angles in an equilateral triangle?"

See also the *Differentiated Instruction Resources* for more strategies.

4.8 EXERCISES

HOMEWORK KEY
○ = See **WORKED-OUT SOLUTIONS** Exs. 5, 17, and 41
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 18, 19, 30, 31, and 46

SKILL PRACTICE

A

1. **VOCABULARY** Define the *vertex angle* of an isosceles triangle.
The angle formed by the legs is the vertex angle.

2. ★ **WRITING** What is the relationship between the base angles of an isosceles triangle? *Explain.* **They are congruent.**

EXAMPLE 1
for Exs. 3–6

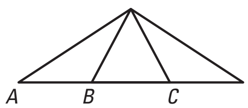
USING DIAGRAMS In Exercises 3–6, use the diagram. Copy and complete the statement. Tell what theorem you used.

3. If $\overline{AE} \cong \overline{DE}$, then $\angle$ __?__ $\cong \angle$ __?__. ***A, D*; Base Angles Theorem**

4. If $\overline{AB} \cong \overline{EB}$, then $\angle$ __?__ $\cong \angle$ __?__. ***A, BEA*; Base Angles Theorem**

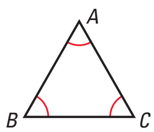
5. If $\angle D \cong \angle CED$, then __?__ $\cong$ __?__. **$\overline{CD}$, $\overline{CE}$; Converse of Base Angles Theorem**

6. If $\angle EBC \cong \angle ECB$, then __?__ $\cong$ __?__.
$\overline{EB}$, $\overline{EC}$; Converse of Base Angles Theorem

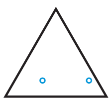


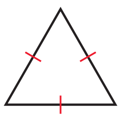
EXAMPLE 2
for Exs. 7–14

REASONING Find the unknown measure.

7. 

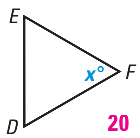
12

8. 
16

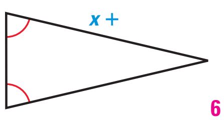
9. 
60°

10. **DRAWING DIAGRAMS** A base angle in an isosceles triangle measures 37°. Draw and label the triangle. What is the measure of the vertex angle?
106°; see margin for art.

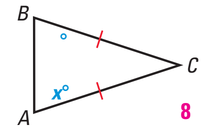
xy ALGEBRA Find the value of *x*.

11. 

20

12. 

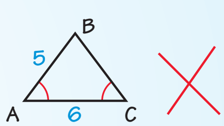
6

13. 

8

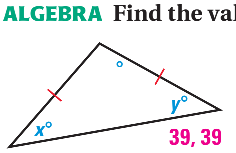
14. **ERROR ANALYSIS** *Describe* and correct the error made in finding *BC* in the diagram shown.
$\overline{AC}$ is not congruent to $\overline{BC}$. $\overline{AB} \cong \overline{BC}$, which makes $BC = 5$.

$\angle A \cong \angle C$, therefore
$\overline{AC} \cong \overline{BC}$. So,
$BC = 6$

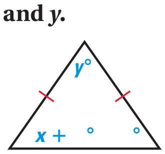


EXAMPLE 3
for Exs. 15–17

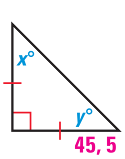
xy ALGEBRA Find the values of *x* and *y*.

15. 

39, 39

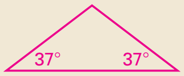
16. 

48, 70

17. 

45, 5

18. ★ **SHORT RESPONSE** Are isosceles triangles always acute triangles? *Explain* your reasoning. **No; an isosceles triangle can have an obtuse or a right vertex angle, which would make it an obtuse or a right triangle.**

10.


4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–14, 19
Day 2:
Exs. 15–18, 20–25, 38–45

Average:
Day 1:
Exs. 1, 2, 4–6, 8–10, 12–14, 19, 26–29
Day 2:
Exs. 16–18, 21–25, 30, 31, 39–48

Advanced:
Day 1:
Exs. 1, 2, 4–6, 8–10, 12–14, 19, 26–29, 35, 36
Day 2:
Exs. 16–18, 22, 24, 30–34, 37*, 40–51*

Block:
Exs. 1, 2, 4–6, 8–10, 12–14, 16–19, 21–31, 39–48

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:
Basic: 3, 8, 15, 38, 42
Average: 4, 10, 16, 39, 42
Advanced: 6, 12, 16, 40, 42

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

Graphing Calculator

Exercise 17 After the students write the system of equations $x = 9y$ and $x + 9y = 90$, have them solve each for y and enter it on the Y= list of their graphing calculator. Graph the lines and find the intersection point, which is the solution of the system.

Teaching Strategy

Exercise 21 You may wish to have students model the figure by using new, unsharpened pencils for the congruent segments. Have students note that while specific numerical values for x and y are not determined by the figure, y can be expressed in terms of x.

Avoiding Common Errors

Exercise 22 Some students may not include -4 as a possible value of x, reasoning that the side lengths of a triangle must be positive. Point out that $3x^2 - 32$ is the expression that is used for a length, not x.

34. 90, about 8.66; one triangle is equiangular, one is isosceles, and the third one is a right triangle. Use the equiangular and isosceles triangles to establish the right triangle and then use the Pythagorean Theorem.

35. 50°, 50°, 80°; 65°, 65°, 50°; there are two distinct exterior angles. If the angle is supplementary to the base angle, the base angles measure 50°. If the angle is supplementary to the vertex angle, then the base angles measure 65°.

36. Since $\angle A$ is the vertex angle of isosceles $\triangle ABC$, $\angle B$ must be congruent to $\angle C$. Since 2 times any angle measure will always be an even number, an even number will be subtracted from 180 to find $m\angle A$. 180 minus an even number will always be an even number, therefore $m\angle A$ must be even.

20. 50, $\frac{1}{2}$; first find y by using the Triangle Sum Theorem followed by the Base Angles Theorem. Next find x by using the Definition of linear pair followed by the Base Angles Theorem. B

21. There is not enough information to find x or y. We need to know the measure of one of the vertex angles.

22. ±4, 4; since $y + 12 = 3x^2 - 32$ and $3x^2 - 32 = 5y - 4$, use the Transitive Property of Equality and set $y + 12 = 5y - 4$ to solve for y and use the value of y to solve for x.

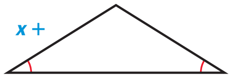
30. Isosceles; two of the angles have the same measure, so two of the sides have the same length by the Converse of the Base Angles Theorem. C

32. 150; one triangle is equiangular and the other two triangles are congruent making $x°$ the measure of the third angle in the center. $x + x + 60 = 360$.

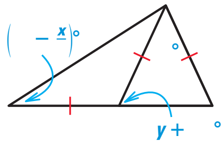
19. ★ **MULTIPLE CHOICE** What is the value of x in the diagram? B

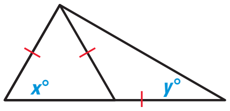
Ⓐ 5 Ⓑ 6

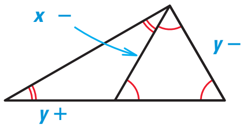
Ⓒ 7 Ⓓ 9



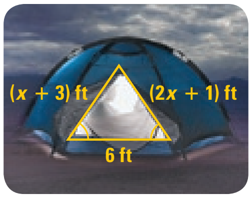
ALGEBRA **Find the values of x and y, if possible. *Explain* your reasoning.** 20–22. See margin.

20. 

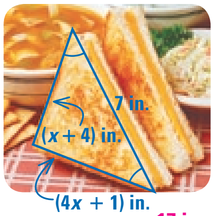
21. 

22. 

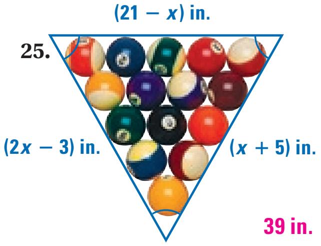
ALGEBRA **Find the perimeter of the triangle.**

23. 

16 ft

24. 

17 in.

25. 

39 in.

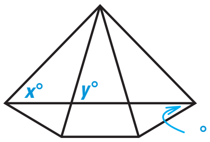
REASONING **In Exercises 26–29, use the diagram. State whether the given values for x, y, and z are possible or not. If not, *explain*.**

26. $x = 90$, $y = 68$, $z = 42$

27. $x = 40$, $y = 72$, $z = 36$ possible

28. $x = 25$, $y = 25$, $z = 15$

29. $x = 42$, $y = 72$, $z = 33$ possible



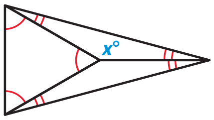
26. Not possible; the isosceles triangle with legs of length 7 cannot contain two 90° angles.

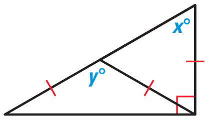
28. Not possible; $x = y$ forms parallel segments, which cannot be two sides of a triangle.

30. ★ **SHORT RESPONSE** In $\triangle DEF$, $m\angle D = (4x + 2)°$, $m\angle E = (6x - 30)°$, and $m\angle F = 3x°$. What type of triangle is $\triangle DEF$? *Explain* your reasoning. See margin.

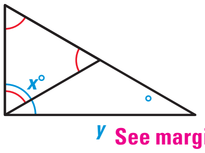
31. ★ **SHORT RESPONSE** In $\triangle ABC$, D is the midpoint of $\overline{AC}$, and $\overline{BD}$ is perpendicular to $\overline{AC}$. *Explain* why $\triangle ABC$ is isosceles. $\triangle ABD \cong \triangle CBD$ by SAS making $\overline{BA} \cong \overline{BC}$ because corresponding parts of congruent triangles are congruent.

ALGEBRA **Find the value(s) of the variable(s). *Explain* your reasoning.**

32. 

33. 

60, 120; solve the system $x + y = 180$ and $180 + 2x - y = 180$.

34. 

See margin.

35. **REASONING** The measure of an exterior angle of an isosceles triangle is 130°. What are the possible angle measures of the triangle? *Explain*. See margin.

36. **PROOF** Let $\triangle ABC$ be isosceles with vertex angle $\angle A$. Suppose $\angle A$, $\angle B$, and $\angle C$ have integer measures. Prove that $m\angle A$ must be even. See margin.

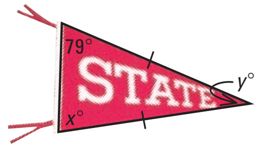
37. **CHALLENGE** The measure of an exterior angle of an isosceles triangle is $x°$. What are the possible angle measures of the triangle in terms of x? *Describe* all the possible values of x. $180 - x, 180 - x, 2x - 180$; $\frac{x}{2}, \frac{x}{2}, 180 - x$; $0 < x < 180$

◯ = See **WORKED-OUT SOLUTIONS** in Student Resources ★ = **STANDARDIZED TEST PRACTICE**

PROBLEM SOLVING

A

38. SPORTS The dimensions of a sports pennant are given in the diagram. Find the values of x and y. **79, 22**



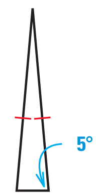
41a. $\angle A$, $\angle ACB$, $\angle CBD$, and $\angle CDB$ are congruent and $\overline{BC} \cong \overline{CB}$ making $\triangle ABC \cong \triangle BCD$ by AAS.

41b. $\triangle ABC$, $\triangle BCD$, $\triangle CDE$, $\triangle DEF$, $\triangle EFG$

41c. $\angle BCD$, $\angle CDE$, $\angle DEF$, $\angle EFG$

39. ADVERTISING A logo in an advertisement is an equilateral triangle with a side length of 5 centimeters. Sketch the logo and give the measure of each side and angle. **See margin.**

40. ARCHITECTURE The Transamerica Pyramid building shown in the photograph has four faces shaped like isosceles triangles. The measure of a base angle of one of these triangles is about 85°. What is the approximate measure of the vertex angle of the triangle? **10°**



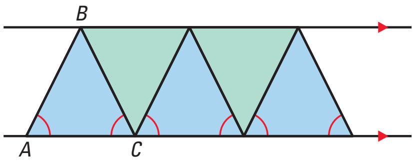
EXAMPLE 4
for Exs. 41–42

41. MULTI-STEP PROBLEM To make a zig-zag pattern, a graphic designer sketches two parallel line segments. Then the designer draws blue and green triangles as shown below. **41a–c. See margin.**

a. Prove that $\triangle ABC \cong \triangle BCD$.

b. Name all the isosceles triangles in the diagram.

c. Name four angles that are congruent to $\angle ABC$.

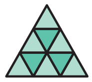
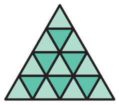


42a. The sides of each new triangle are the sum of the same number of congruent segments.

42b. 1 square unit, 4 square units, 9 square units, 16 square units

42c. $1^2, 2^2, 3^2 \ldots$; 49 square units; the numbers representing the areas are the sequence of perfect squares.

42. ★ VISUAL REASONING In the pattern below, each small triangle is an equilateral triangle with an area of 1 square unit.

Triangle				
Area	1 square unit	?	?	?

a. **Reasoning** *Explain* how you know that any triangle made out of equilateral triangles will be an equilateral triangle.

b. **Area** Find the areas of the first four triangles in the pattern.

c. **Make a Conjecture** *Describe* any patterns in the areas. Predict the area of the seventh triangle in the pattern. *Explain* your reasoning.

B

43. REASONING Let $\triangle PQR$ be an isosceles right triangle with hypotenuse $\overline{QR}$. Find $m\angle P$, $m\angle Q$, and $m\angle R$. **90°, 45°, 45°**

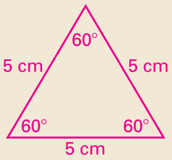
44. REASONING *Explain* how the Corollary to the Base Angles Theorem follows from the Base Angles Theorem. **If a triangle is equilateral it is also isosceles, using these two facts it can be shown that the triangle is equiangular.**

45. PROVING THEOREM 4.8 Write a proof of the Converse of the Base Angles Theorem. **See margin.**

Mathematical Reasoning

Exercise 37 Suggest that students sketch an isosceles triangle and indicate congruent sides with hash marks. Have them draw an exterior angle at each vertex. Then have them consider which exterior angle to label $x°$. This should help them see that there is more than one possibility.

39.



45. Statements (Reasons)

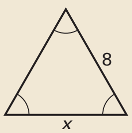
1. $\triangle ABC$ with $\angle B \cong \angle C$ (Given)
2. Draw the perpendicular line from A to $\overline{BC}$ and label its intersection with $\overline{BC}$ as D. (Perpendicular Postulate)
3. $\angle ADB$ and $\angle ADC$ are right angles. (If two lines are $\perp$, then they form 4 right angles.)
4. $\angle ADB \cong \angle ADC$ (Right Angles Congruence Theorem)
5. $\overline{AD} \cong \overline{AD}$ (Reflexive Property of Congruence)
6. $\triangle ADB \cong \triangle ADC$ (AAS)
7. $\overline{AB} \cong \overline{AC}$ (Corr. parts of $\cong$ ⧋ are $\cong$.)

5 ASSESS AND RETEACH

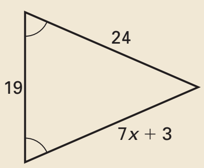
Daily Homework Quiz

Also available online

Find the value of x.

1. 

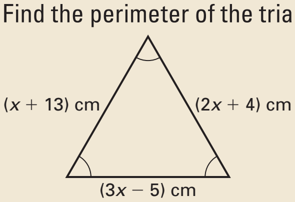
8

2. 

3

3. If the measure of the vertex angle of an isosceles triangle is 112°, what are the measures of the base angles? **34°, 34°**

4. Find the perimeter of the triangle.



66 cm

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

46a. Statements (Reasons)

1. $\overline{AB} \cong \overline{CD}$, $\overline{AE} \cong \overline{DE}$, $\angle BAE \cong \angle CDB$ (Given)

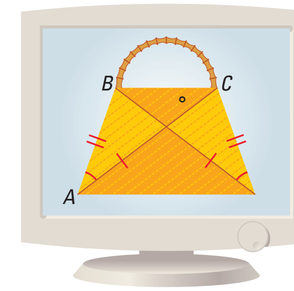
2. $\triangle ABE \cong \triangle DCE$ (SAS)

46b. $\triangle AED$, $\triangle BEC$

46c. $\angle EDA$, $\angle EBC$, $\angle ECB$

49, 50. See Additional Answers.

46. ★ **EXTENDED RESPONSE** Sue is designing fabric purses that she will sell at the school fair. Use the diagram of one of her purses. **a–c. See margin.**



a. Prove that $\triangle ABE \cong \triangle DCE$.

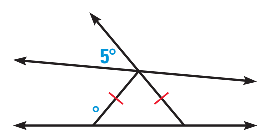
b. Name the isosceles triangles in the purse.

c. Name three angles that are congruent to $\angle EAD$.

d. What If? If the measure of $\angle BEC$ changes, does your answer to part (c) change? *Explain.*
No; $\triangle AED$ and $\triangle BEC$ remain isosceles triangles with $\angle BEC \cong \angle AED$.

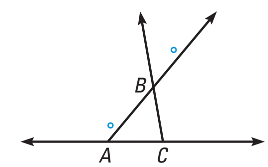
REASONING FROM DIAGRAMS **Use the information in the diagram to answer the question. *Explain* your reasoning.**

47. Is $p \parallel q$?



No; $m\angle 1 = 50°$, so $m\angle 2 = 50°$ and corresponds to the angle measuring 45°, therefore p is not parallel to q.

48. Is $\triangle ABC$ isosceles?

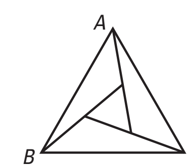


Yes; $m\angle ABC = 50°$ and $m\angle BAC = 50°$. The Converse of Base Angles Theorem guarantees that $\overline{AC} \cong \overline{BC}$ making $\triangle ABC$ isosceles.

C **49. PROOF** Write a proof. **See margin.**

GIVEN ▶ $\triangle ABC$ is equilateral, $\angle CAD \cong \angle ABE \cong \angle BCF$.

PROVE ▶ $\triangle DEF$ is equilateral.



50. COORDINATE GEOMETRY The coordinates of two vertices of $\triangle TUV$ are $T(0, 4)$ and $U(4, 0)$. *Explain* why the triangle will always be an isosceles triangle if V is any point on the line $y = x$ except $(2, 2)$. **See margin.**

51. CHALLENGE The lengths of the sides of a triangle are $3t$, $5t - 12$, and $t + 20$. Find the values of t that make the triangle isosceles. *Explain.*
6, 8, 10; set $3t = 5t - 12$, $3t = t + 20$, $5t - 12 = t + 20$ and solve for t.

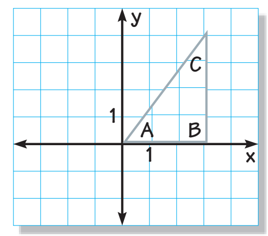
Investigating Geometry ACTIVITY *Use before Perform Congruence Transformations*

Investigate Slides and Flips

MATERIALS • graph paper • pencil

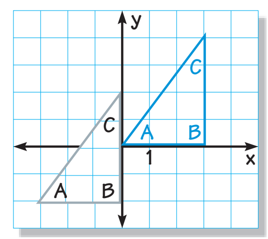
QUESTION **What happens when you slide or flip a triangle?**

EXPLORE 1 **Slide a triangle**



STEP 1 ***Draw a triangle*** Draw a scalene right triangle with legs of length 3 units and 4 units on a piece of graph paper. Cut out the triangle.

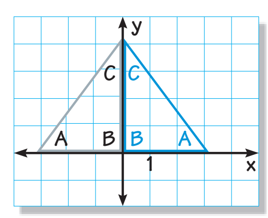
STEP 2 ***Draw coordinate plane*** Draw axes on the graph paper. Place the cut-out triangle so that the coordinates of the vertices are integers. Trace around the triangle and label the vertices.



STEP 3 ***Slide triangle*** Slide the cut-out triangle so it moves left and down. Write a description of the *transformation* and record ordered pairs in a table like the one shown. Repeat this step three times, sliding the triangle left or right *and* up or down to various places in the coordinate plane.

Slide 3 units left and 2 units down.		
Vertex	**Original position**	**New position**
A	(0, 0)	(−3, −2)
B	(3, 0)	(0, −2)
C	(3, 4)	(0, 2)

EXPLORE 2 **Flip a triangle**



STEP 1 ***Draw a coordinate plane*** Draw and label a second coordinate plane. Place the cut-out triangle so that one vertex is at the origin and one side is along the *y*-axis, as shown.

STEP 2 ***Flip triangle*** Flip the cut-out triangle over the *y*-axis. Record a description of the *transformation* and record the ordered pairs in a table. Repeat this step, flipping the triangle over the *x*-axis.

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. How are the coordinates of the original position of the triangle related to the new position in a slide? in a flip? **See margin.**

2. Is the original triangle congruent to the new triangle in a slide? in a flip? *Explain* your reasoning. **Yes; yes; they remain the same size and shape.**

1. The *x*-coordinate has been increased or decreased by the direction and magnitude of the horizontal movement while the *y*-coordinate has been increased or decreased by the direction and magnitude of the vertical movement; the coordinates of *B*, *C*, and the *y*-coordinate of *A* remain the same. The *x*-coordinate of *A* is the opposite of the original *x*-coordinate.

1 PLAN AND PREPARE

Explore the Concept

- Students will slide and flip a triangle.
- This activity leads into the study of transformations in this lesson.

Materials

Each student will need:

- graph paper
- scissors

Recommended Time

Work activity: 10 min

Discuss results: 5 min

Grouping

Students should work individually.

2 TEACH

Tips for Success

For the activity, students can cut the triangle they are going to move from colored card stock.

Key Questions

- How can you obtain algebraically the coordinates of the image after a slide? **Add or subtract from *x*- and *y*-coordinates as indicated.**
- Reflect the triangle over the *y*-axis without putting one side on the *y*-axis. How are the coordinates of the new triangle related to those of the original? ***x*-coordinates are opposites, *y*-coordinates are equal.**

Key Discovery

A triangle obtained by a slide or a flip is congruent to the original triangle.

3 ASSESS AND RETEACH

1. Explain how you can obtain the coordinates of the new triangle after a flip over the *x*-axis or *y*-axis. **Take the opposite of the *y*- or *x*-coordinates.**

1 PLAN AND PREPARE

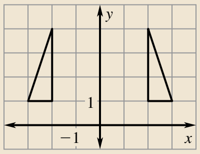
Warm-Up Exercises

Also available online

1. Find the length of $\overline{AB}$ for $A(2, 7)$ and $B(7, -5)$. **13**

2. What point is 6 units to the right of (3, 5)? **(9, 5)**

3. Are these triangles congruent?



Yes, by the SSS Cong. Post. or by the SAS Cong. Post.

Notetaking Guide

Available online

Promotes interactive learning and notetaking skills.

Pacing

Basic: 2 days
Average: 2 days
Advanced: 2 days
Block: 1 block

• See *Teaching Guide/Lesson Plan.*

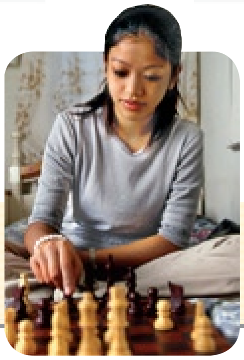
2 FOCUS AND MOTIVATE

Essential Question

Big Idea 3

What transformations create an image congruent to the original figure? **Tell students they will learn how to answer this question by learning about reflections, rotations, and translations.**

4.9 Perform Congruence Transformations



Before You determined whether two triangles are congruent.

Now You will create an image congruent to a given triangle.

Why So you can describe chess moves, as in Ex. 41.

Key Vocabulary
- **transformation**
- **image**
- **translation**
- **reflection**
- **rotation**
- **congruence transformation**

A **transformation** is an operation that moves or changes a geometric figure in some way to produce a new figure. The new figure is called the A transformation can be shown using an arrow.

The order of the vertices in the transformation statement tells you that P is the image of A, Q is the image of B, and R is the image of C.

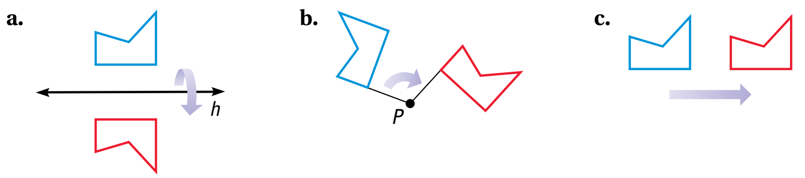
$$\triangle ABC \rightarrow \triangle PQR$$

Original figure — Image

There are three main types of transformations. A **translation** moves every point of a figure the same distance in the same direction. A **reflection** uses a *line of reflection* to create a mirror image of the original figure. A **rotation** turns a figure about a fixed point, called the *center of rotation*.

EXAMPLE 1 Identify transformations

Name the type of transformation demonstrated in each picture.



Reflection in a horizontal line — Rotation about a point — Translation in a straight path

✓ GUIDED PRACTICE for Example 1

1. Name the type of transformation shown. **reflection**

CONGRUENCE Translations, reflections, and rotations are three types of *congruence transformations*. A **congruence transformation** changes the position of the figure without changing its size or shape.

TRANSLATIONS In a coordinate plane, a translation moves an object a given distance right or left and up or down. You can use coordinate notation to describe a translation.

READ DIAGRAMS
In this book, the original figure is blue and the transformation of the figure is red, unless otherwise stated.

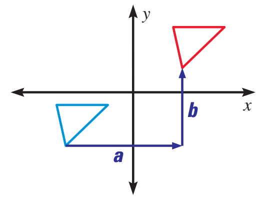
KEY CONCEPT *For Your Notebook*

Coordinate Notation for a Translation

You can describe a translation by the notation

$(x, y) \rightarrow (x + a, y + b)$

which shows that each point (x, y) of the blue figure is translated horizontally a units and vertically b units.



EXAMPLE 2 Translate a figure in the coordinate plane

Figure *ABCD* has the vertices $A(-4, 3)$, $B(-2, 4)$, $C(-1, 1)$, and $D(-3, 1)$. Sketch *ABCD* and its image after the translation $(x, y) \rightarrow (x + 5, y - 2)$.

Solution

First draw *ABCD*. Find the translation of each vertex by adding 5 to its x-coordinate and subtracting 2 from its y-coordinate. Then draw *ABCD* and its image.

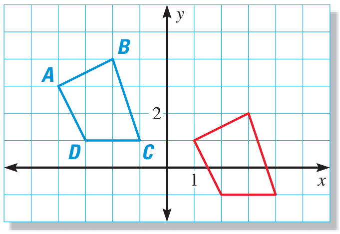
$(x, y) \rightarrow (x + 5, y - 2)$

$A(-4, 3) \rightarrow (1, 1)$

$B(-2, 4) \rightarrow (3, 2)$

$C(-1, 1) \rightarrow (4, -1)$

$D(-3, 1) \rightarrow (2, -1)$

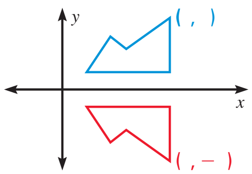


REFLECTIONS In this lesson, when a reflection is shown in a coordinate plane, the line of reflection is always the x-axis or the y-axis.

KEY CONCEPT *For Your Notebook*

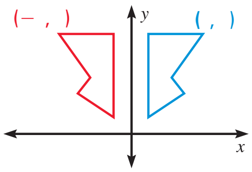
Coordinate Notation for a Reflection

Reflection in the x-axis



Multiply the y-coordinate by -1.
$(x, y) \rightarrow (x, -y)$

Reflection in the y-axis



Multiply the x-coordinate by -1.
$(x, y) \rightarrow (-x, y)$

Motivating the Lesson

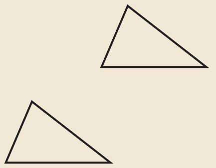
Have students consider drawing a pattern that is symmetric about a vertical line. They could draw one side of the pattern and then reflect it over the line to get the other half. Tell students they will study several transformations that keep figures the same size and shape. These are used in many fields including art.

3 TEACH

Extra Example 1

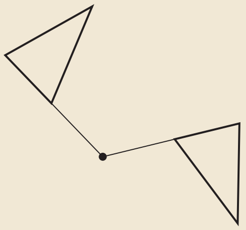
Name the type of transformation demonstrated in each picture.

a.



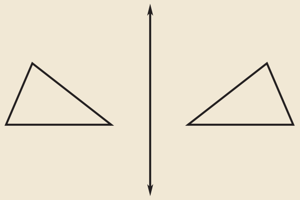
translation in a straight path

b.



rotation about a point

c.



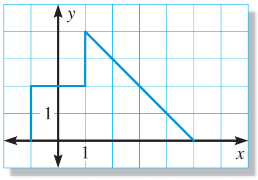
reflection in a vertical line

Key Question to Ask for Example 1

- Is the image figure in each case congruent to the original figure? Explain. **Yes, they have the same size and shape.**

EXAMPLE 3 Reflect a figure in the *x*-axis

WOODWORK **You are drawing a pattern for a wooden sign. Use a reflection in the x-axis to draw the other half of the pattern.**



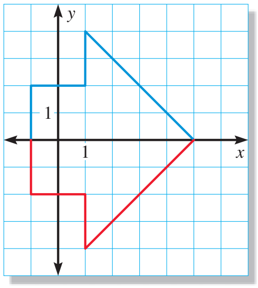
Solution

Multiply the y-coordinate of each vertex by -1 to find the corresponding vertex in the image.

$$(x, y) \rightarrow (x, -y)$$

$(-1, 0)$	$\rightarrow$	$(-1, 0)$	$(-1, 2)$	$\rightarrow$	$(-1, -2)$
$(1, 2)$	$\rightarrow$	$(1, -2)$	$(1, 4)$	$\rightarrow$	$(1, -4)$
$(5, 0)$	$\rightarrow$	$(5, 0)$			

Use the vertices to draw the image. You can check your results by looking to see if each original point and its image are the same distance from the x-axis.



Animated Geometry at my.hrw.com

✓ GUIDED PRACTICE for Examples 2 and 3

2. The vertices of $\triangle ABC$ are $A(1, 2)$, $B(0, 0)$, and $C(4, 0)$. A translation of $\triangle ABC$ results in the image $\triangle DEF$ with vertices $D(2, 1)$, $E(1, -1)$, and $F(5, -1)$. *Describe* the translation in words and in coordinate notation.

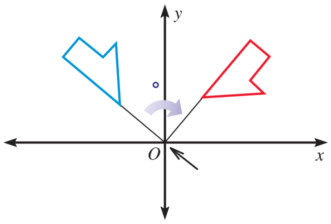
2. Add one to each x-coordinate and subtract one from each y-coordinate, $(x, y) \rightarrow (x + 1, y - 1)$.

3. The endpoints of $\overline{RS}$ are $R(4, 5)$ and $S(1, -3)$. A reflection of $\overline{RS}$ results in the image $\overline{TU}$, with coordinates $T(4, -5)$ and $U(1, 3)$. Tell which axis $\overline{RS}$ was reflected in and write the coordinate rule for the reflection.
x-axis, $(x, y) \rightarrow (x, -y)$

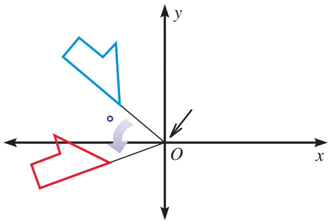
ROTATIONS In this lesson, if a rotation is shown in a coordinate plane, the center of rotation is the origin.

The direction of rotation can be either *clockwise* or *counterclockwise*. The *angle of rotation* is formed by rays drawn from the center of rotation through corresponding points on the original figure and its image.

90° clockwise rotation



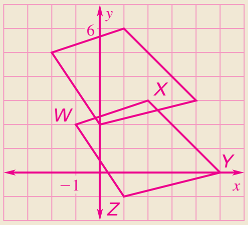
60° counterclockwise rotation



Notice that rotations preserve distances from the center of rotation. So, segments drawn from the center of rotation to corresponding points on the figures are congruent.

Extra Example 2

Figure $WXYZ$ has the vertices $W(-1, 2)$, $X(2, 3)$, $Y(5, 0)$, and $Z(1, -1)$. Sketch $WXYZ$ and its image after the translation $(x, y) \rightarrow (x - 1, y + 3)$.

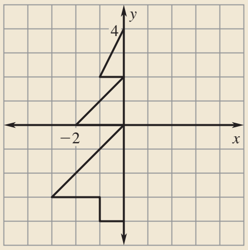


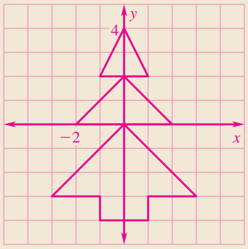
Key Questions to Ask for Example 2

- In the example, the orientation of $ABCD$ is clockwise. What is the orientation of the image? **clockwise**
- Does a translation keep the orientation the same? **yes**

Extra Example 3

You are drawing a pattern for a cross stitch design. Use a reflection in the y-axis to draw the other half of the pattern.





Key Question to Ask for Example 3

- Does a reflection preserve orientation? **No, it reverses it.**

Animated Geometry my.hrw.com

An **Animated Geometry** activity is available online for **Example 3**. This activity is also part of **Power Presentations**.

Differentiated Instruction

English Learners Students learning English may confuse the words *transformation*, *translation*, *reflection*, and *rotation*. Explain that a transformation refers to a translation, reflection, or rotation. *Transform* means "to change." Help students remember each transformation by thinking about these definitions: *translate* means "to move something," *reflect* means "to show a mirror image," and *rotate* means "to turn around a point."

See also the *Differentiated Instruction Resources* for more strategies.

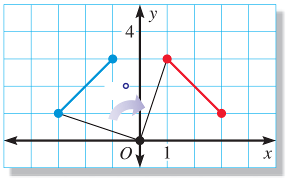
EXAMPLE 4 Identify a rotation

Graph $\overline{AB}$ and $\overline{CD}$. Tell whether $\overline{CD}$ is a rotation of $\overline{AB}$ about the origin. If so, give the angle and direction of rotation.

a. $A(-3, 1)$, $B(-1, 3)$, $C(1, 3)$, $D(3, 1)$ **b.** $A(0, 1)$, $B(1, 3)$, $C(-1, 1)$, $D(-3, 2)$

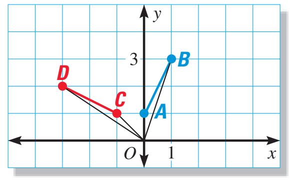
Solution

a.



$m\angle AOC = m\angle BOD = 90°$
This is a 90° clockwise rotation.

b.

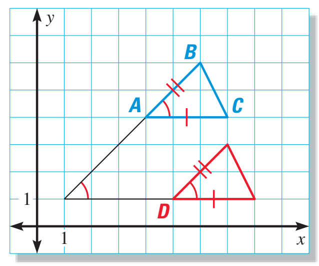


$m\angle AOC < m\angle BOD$
This is not a rotation.

EXAMPLE 5 Verify congruence

The vertices of $\triangle ABC$ are $A(4, 4)$, $B(6, 6)$, and $C(7, 4)$. The notation $(x, y) \to (x + 1, y - 3)$ describes the translation of $\triangle ABC$ to $\triangle DEF$. Show that $\triangle ABC \cong \triangle DEF$ to verify that the translation is a congruence transformation.

Solution



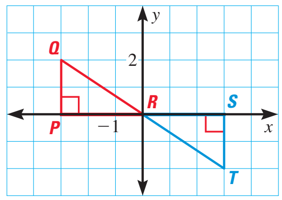
S You can see that $AC = DF = 3$, so $\overline{AC} \cong \overline{DF}$.

A Using the slopes, $\overline{AB} \parallel \overline{DE}$ and $\overline{AC} \parallel \overline{DF}$. If you extend $\overline{AB}$ and $\overline{DF}$ to form $\angle G$, the Corresponding Angles Postulate gives you $\angle BAC \cong \angle G$ and $\angle G \cong \angle EDF$. Then, $\angle BAC \cong \angle EDF$ by the Transitive Property of Congruence.

S Using the Distance Formula, $AB = DE = 2\sqrt{2}$ so $\overline{AB} \cong \overline{DE}$. So, $\triangle ABC \cong \triangle DEF$ by the SAS Congruence Postulate.

▸ Because $\triangle ABC \cong \triangle DEF$, the translation is a congruence transformation.

✓ GUIDED PRACTICE for Examples 4 and 5



4. Tell whether $\triangle PQR$ is a rotation of $\triangle STR$. If so, give the angle and direction of rotation.
yes; 180° counterclockwise

5. Show that $\triangle PQR \cong \triangle STR$ to verify that the transformation is a congruence transformation.
$\overline{PQ} \cong \overline{ST}$, $\overline{PR} \cong \overline{SR}$, by HL, $\triangle PQR \cong \triangle STR$ so it is a congruence transformation.

Vocabulary

Have students develop a method to remember what a reflection, rotation, and translation are. For example, it might help to observe that the words "flip" and "reflection" both contain the letter combination "fl". The words "translation" and "slide" both contain the letter combination "sl".

Extra Example 4

Graph $\overline{PQ}$ and $\overline{RS}$. Tell whether $\overline{RS}$ is a rotation of $\overline{PQ}$ about the origin. If so, give the angle and direction of rotation.

a. $P(2, 6)$, $Q(5, 1)$, $R(6, -1)$, $S(1, -2)$
not rotation

b. $P(4, 2)$, $Q(3, 3)$, $R(-2, 4)$, $S(-3, 3)$
rotation 90° counterclockwise

Extra Example 5

The vertices of $\triangle DEF$ are $D(-1, 3)$, $E(4, 2)$, and $F(1, -2)$. The rule $(x, y) \to (x - 2, y + 4)$ was used to translate $\triangle DEF$ to $\triangle XYZ$. Show that $\triangle DEF \cong \triangle XYZ$ to verify that the translation is a congruence transformation. **$DE = XY = \sqrt{26}$; $DF = XZ = \sqrt{29}$; $EF = YZ = 5$. $\triangle DEF \cong \triangle XYZ$ by the SSS Congruence Postulate.**

Key Question to Ask for Example 5

- How else could you have proved the triangles congruent? **Find the lengths of all three sides of both triangles and use SSS.**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: What transformations create an image congruent to the original figure?

- **A translation is a transformation that moves every point of a figure the same distance in the same direction.**
- **A reflection is a transformation that uses a line of reflection to create a mirror image of the original figure.**
- **A rotation is a transformation in which a figure is turned about a fixed point.**

Translations, reflections, and rotations create an image congruent to the original figure.

4 PRACTICE AND APPLY

Assignment Guide

Answers for all exercises available online

Basic:
Day 1:
Exs. 1–16
Day 2:
Exs. 17–29, 38–42

Average:
Day 1:
Exs. 1, 2, 4, 8, 10–16 even, 26–31
Day 2:
Exs. 17–25, 32–36, 38–43

Advanced:
Day 1:
Exs. 1, 2, 4–16 even, 26–31, 37*
Day 2:
Exs. 17–25, 32–36, 38–44*

Block:
Exs. 1, 2, 4, 8, 10–16 even, 17–36, 38–43

Differentiated Instruction

See *Differentiated Instruction Resources* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:

Basic: 4, 10, 17, 20, 38
Average: 8, 12, 18, 21, 38
Advanced: 8, 14, 19, 22, 42

Extra Practice

- Student Edition
- Chapter Resource Book: Practice levels A, B, C

Practice Worksheet

An easily-readable reduced practice page can be found at the beginning of this chapter.

4.9 EXERCISES

HOMEWORK KEY
○ = See **WORKED-OUT SOLUTIONS** Exs. 11, 23, and 39
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 25, 40, 41, and 43

SKILL PRACTICE

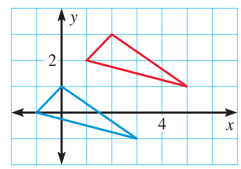
A

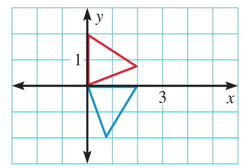
1. **VOCABULARY** *Describe* the translation $(x, y) \rightarrow (x - 1, y + 4)$ in words.
Subtract one from each *x*-coordinate and add 4 to each *y*-coordinate.

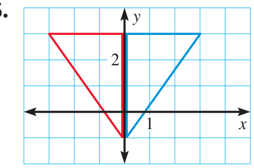
2. ★ **WRITING** *Explain* why the term *congruence transformation* is used in describing translations, reflections, and rotations.
The image is congruent to the original figure.

EXAMPLE 1 for Exs. 3–8

IDENTIFYING TRANSFORMATIONS Name the type of transformation shown.

3. 
translation

4. 
rotation

5. 
reflection

WINDOWS Decide whether the moving part of the window is a translation.

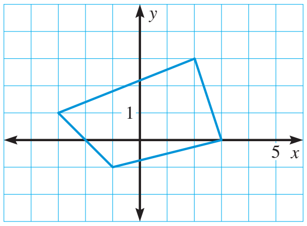
6. Double hung **yes**


7. Casement **no**


8. Sliding **yes**


EXAMPLE 2 for Exs. 9–16

DRAWING A TRANSLATION Copy figure *ABCD* and draw its image after the translation. **9–12. See margin.**



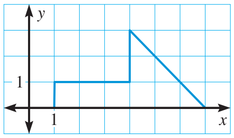
9. $(x, y) \rightarrow (x + 2, y - 3)$
10. $(x, y) \rightarrow (x - 1, y - 5)$
11. $(x, y) \rightarrow (x + 4, y + 1)$
12. $(x, y) \rightarrow (x - 2, y + 3)$

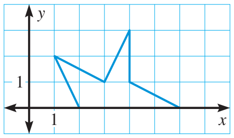
COORDINATE NOTATION Use coordinate notation to *describe* the translation.

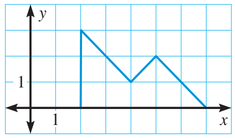
13. 4 units to the left, 2 units down **$(x, y) \rightarrow (x - 4, y - 2)$**
14. 6 units to the right, 3 units up **$(x, y) \rightarrow (x + 6, y + 3)$**
15. 2 units to the right, 1 unit down **$(x, y) \rightarrow (x + 2, y - 1)$**
16. 7 units to the left, 9 units up **$(x, y) \rightarrow (x - 7, y + 9)$**

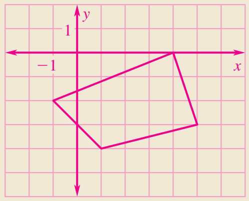
EXAMPLE 3 for Exs. 17–19

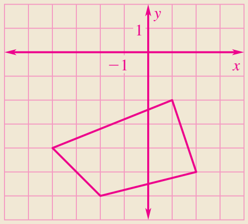
DRAWING Use a reflection in the *x*-axis to draw the other half of the figure. **17–19. See margin.**

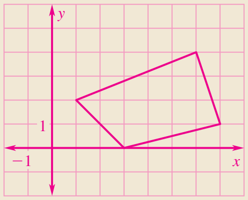
17. 

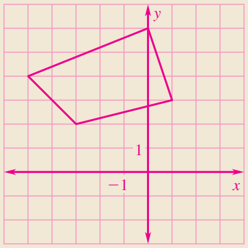
18. 

19. 

9. 

10. 

11. 

12. 

EXAMPLE 4 for Exs. 20–23

ROTATIONS **Use the coordinates to graph $\overline{AB}$ and $\overline{CD}$. Tell whether $\overline{CD}$ is a rotation of $\overline{AB}$ about the origin. If so, give the angle and direction of rotation.** **20–23. See margin for art.**

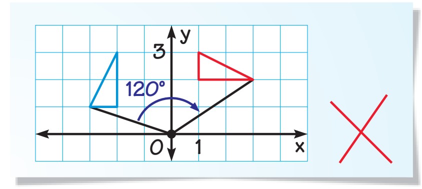
20. $A(1, 2), B(3, 4), C(2, -1), D(4, -3)$ **rotation; 90° clockwise**

21. $A(-2, -4), B(-1, -2), C(4, 3), D(2, 1)$ **not a rotation**

22. $A(-4, 0), B(-4, -4), C(4, 4), D(0, 4)$ **not a rotation**

23. $A(1, 2), B(3, 0), C(2, -1), D(2, -3)$ **not a rotation**

24. **ERROR ANALYSIS** A student says that the red triangle is a 120° clockwise rotation of the blue triangle about the origin. *Describe* and correct the error.
The red triangle rotation segment should connect corresponding angles of the triangle; the red triangle is rotated 90° clockwise.



B 25. ★ **WRITING** Can a point or a line segment be its own image under a transformation? *Explain* and illustrate your answer.
Yes; take any point or any line segment and rotate 360°; see margin for art.

APPLYING TRANSLATIONS **Complete the statement using the description of the translation. In the description, points (0, 3) and (2, 5) are two vertices of a hexagon.**

26. If (0, 3) translates to (0, 0), then (2, 5) translates to __?__. **(2, 2)**

27. If (0, 3) translates to (1, 2), then (2, 5) translates to __?__. **(3, 4)**

28. If (0, 3) translates to (−3, −2), then (2, 5) translates to __?__. **(−1, 0)**

ALGEBRA **A point on an image and a transformation are given. Find the corresponding point on the original figure.**

29. Point on image: (4, 0); transformation: $(x, y) \rightarrow (x + 2, y - 3)$ **(2, 3)**

30. Point on image: (−3, 5); transformation: $(x, y) \rightarrow (-x, y)$ **(3, 5)**

31. Point on image: (6, −9); transformation: $(x, y) \rightarrow (x - 7, y - 4)$ **(13, −5)**

32. **CONGRUENCE** Show that the transformation in Exercise 3 is a congruence transformation.
The corresponding sides of each triangle are congruent therefore the triangles are congruent.

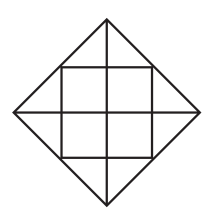
DESCRIBING AN IMAGE **State the segment or triangle that represents the image. You can use tracing paper to help you see the rotation.**

33. 90° clockwise rotation of $\overline{ST}$ about E **$\overline{UV}$**

34. 90° counterclockwise rotation of $\overline{BX}$ about E **$\overline{AV}$**

35. 180° rotation of $\triangle BWX$ about E **$\triangle DST$**

36. 180° rotation of $\triangle TUA$ about E **$\triangle XYC$**



C 37. **CHALLENGE** Solve for the variables in the transformation of $\overline{AB}$ to $\overline{CD}$ and then to $\overline{EF}$. **$m = 3, n = 11, a = 2, g = 2, h = \frac{1}{4}$**

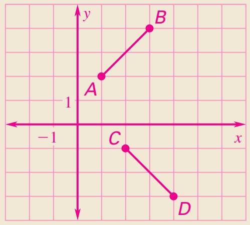
$A(2, 3)$, $B(4, 2a)$ → Translation: $(x, y) \rightarrow (x - 2, y + 1)$ → $C(m - 3, 4)$, $D(n - 9, 5)$ → Reflection: in x-axis → $E(0, g - 6)$, $F(8h, -5)$

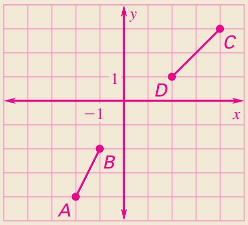
Avoiding Common Errors

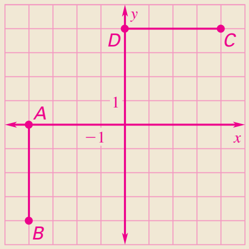
Exercises 9–12 Students may reverse the effects on x and y in the rule. For instance, in Exercise 9 they may move up 2 and left 3. Review the idea that a change in the x-coordinate corresponds to a change in horizontal position and a change in the y-coordinate corresponds to a change in vertical position.

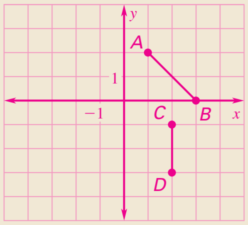
Study Strategy

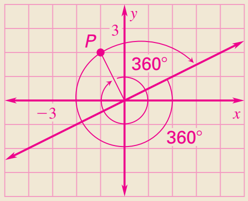
Exercises 29–31 Suggest that students graph the point they are given and work backwards. Since the translation in Exercise 29 moves points 2 units to the right and 3 units down, start with the final point and move 2 units to the left and up 3 units to find the original point.

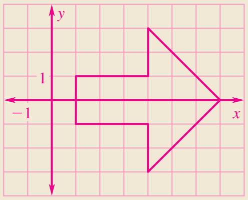
20.


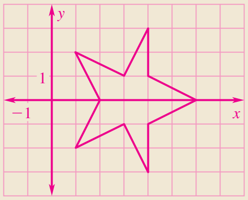
21.


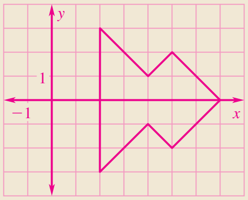
22.


23.


25.


17.


18.


19.


Mathematical Reasoning

Exercise 39 Ask students how they could use reflections instead of rotations to move the stencil from *A* to *B* and from *A* to *C*. **Reflect in the *y*-axis and then reflect in the *x*-axis.**

Internet Reference

Exercise 41 Additional information about playing the game of chess can be found at www.uschess.org

Teaching Strategy

Exercise 43 If students have difficulty visualizing the resulting figure, let them use a piece of paper and scissors, do the cuts and compare the result to the choices.

40a. MOM TOT

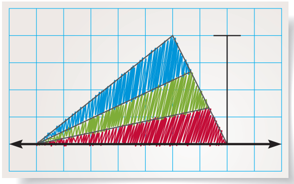
40b. HI OH

42. The slopes of $\overline{BC}$ and $\overline{EF}$ are both -1 and the slopes of $\overline{AB}$ and $\overline{DE}$ are both 1. Therefore, $\overline{AB} \perp \overline{BC}$ since the product of their slopes is -1. Similarly, $\overline{DE} \perp \overline{EF}$. Thus $\triangle ABC$ and $\triangle DEF$ are right triangles. Using the distance formula, $BC = EF = \sqrt{2}$ and $AC = DF = \sqrt{10}$. So, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$. By the HL Congruence Theorem, $\triangle ABC \cong \triangle DEF$. Therefore, $\triangle DEF$ is a congruence transformation of $\triangle ABC$, described by the notation $(x, y) \to (x - 5, y + 1)$.

PROBLEM SOLVING

EXAMPLE 3 A for Ex. 38

38. KITES The design for a kite shows the layout and dimensions for only half of the kite.

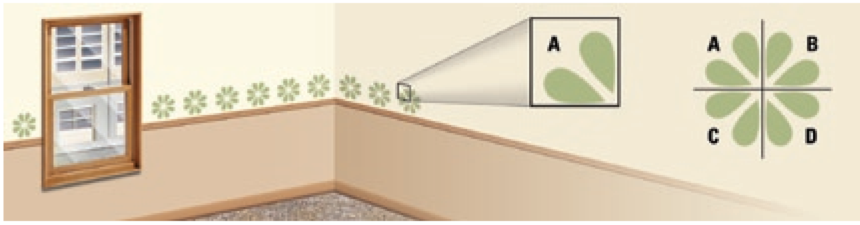


a. What type of transformation can a designer use to create plans for the entire kite?

38a. The designer can reflect the layout over the horizontal line.

b. What is the maximum width of the entire kite? **4 ft**

39. STENCILING You are stenciling a room in your home. You want to use the stencil pattern below on the left to create the design shown. Give the angles and directions of rotation you will use to move the stencil from *A* to *B* and from *A* to *C*. **90° clockwise, 90° counterclockwise**



40. ★ OPEN-ENDED MATH Some words reflect onto themselves through a vertical line of reflection. An example is shown. **40a, b. See margin for art.**

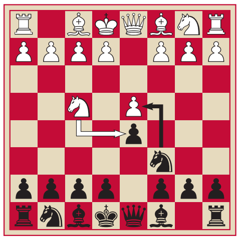
WOW

a. Find two other words with vertical lines of reflection. Draw the line of reflection for each word. ***Sample answer:* MOM, TOT**

b. Find two words with horizontal lines of reflection. Draw the line of reflection for each word. ***Sample answer:* HI, OH**

B

41. ★ SHORT RESPONSE In chess, six different kinds of pieces are moved according to individual rules. The Knight (shaped like a horse) moves in an "L" shape. It moves two squares horizontally or vertically and then one additional square perpendicular to its original direction. When a knight lands on a square with another piece, it *captures* that piece.



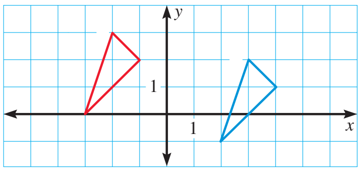
a. *Describe* the translation used by the Black Knight to capture the White Pawn. **$(x, y) \to (x - 1, y + 2)$**

b. *Describe* the translation used by the White Knight to capture the Black Pawn. **$(x, y) \to (x + 2, y - 1)$**

c. After both pawns are captured, can the Black Knight capture the White Knight? *Explain.* **No; the translation needed does not match a knight's move.**

EXAMPLE 5 for Ex. 42

42. VERIFYING CONGRUENCE Show that $\triangle ABC$ and $\triangle DEF$ are right triangles and use the HL Congruence Theorem to verify that $\triangle DEF$ is a congruence transformation of $\triangle ABC$. **See margin.**

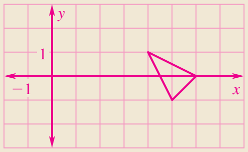


○ = See **WORKED-OUT SOLUTIONS** in Student Resources

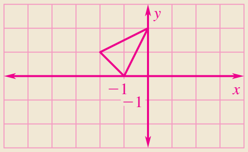
★ = **STANDARDIZED TEST PRACTICE**

Quiz in next page

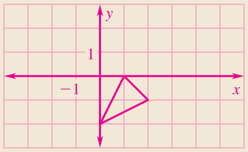
4.



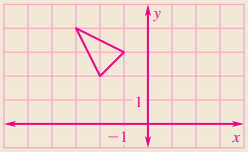
5.



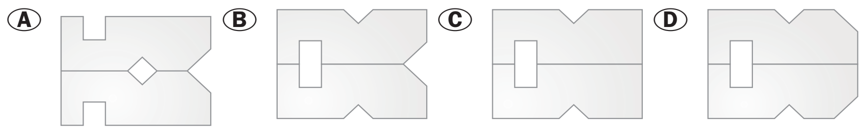
6.



7.



43. ★ **MULTIPLE CHOICE** A piece of paper is folded in half and some cuts are made, as shown. Which figure represents the unfolded piece of paper? **B**

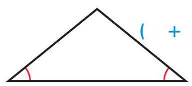


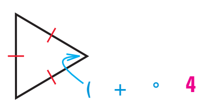
Ⓐ Ⓑ Ⓒ Ⓓ

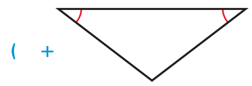
C 44. **CHALLENGE** A triangle is rotated 90° counterclockwise and then translated three units up. The vertices of the final image are $A(-4, 4)$, $B(-1, 6)$, and $C(-1, 4)$. Find the vertices of the original triangle. Would the final image be the same if the original triangle was translated 3 units up and then rotated 90° counterclockwise? *Explain* your reasoning.
(1, 4), (1, 1), (3, 1); no; the final image would have a different rotation segment.

QUIZ

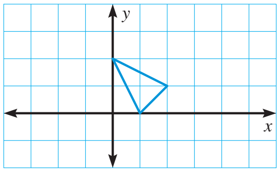
Find the value of x.

1.  **2**

2.  **4**

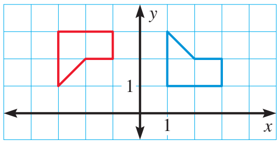
3.  **5**

Copy $\triangle EFG$ and draw its image after the transformation. Identify the type of transformation. **4–7. See margin for art.**



4. $(x, y) \to (x + 4, y - 1)$ **translation**
5. $(x, y) \to (-x, y)$ **reflection in the y-axis**
6. $(x, y) \to (x, -y)$ **reflection in the x-axis**
7. $(x, y) \to (x - 3, y + 2)$ **translation**

8. Is Figure B a rotation of Figure A about the origin? If so, give the angle and direction of rotation. **no**



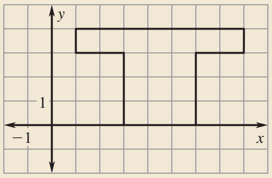
5 ASSESS AND RETEACH

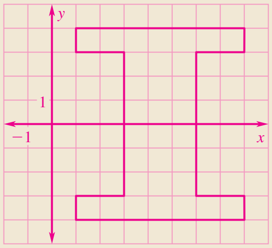
Daily Homework Quiz

Also available online

1. Use coordinate notation to describe the translation 3 units to the left and 1 unit up.
$(x, y) \to (x - 3, y + 1)$

2. Use a reflection in the x-axis to draw the other half of the figure.





3. Tell whether $\overline{XY}$ is a rotation of $\overline{GH}$ about the origin if the points are $X(-6, 2)$, $Y(-4, 3)$, $G(6, -2)$, and $H(4, -3)$. If so, give the angle and direction of rotation. **Yes; 180° clockwise or counterclockwise**

Online Quiz

Available at **my.hrw.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter Resource Book
- Study Guide in Chapter Resource Book
- Practice Workbook
- @HomeTutor

Challenge

Additional challenge is available in the Chapter Resource Book.

Quiz

An easily-readable reduced copy of the quiz from the Assessment Book can be found at the beginning of this chapter.

3. the length of the side forming the 34° angle with side measuring 8 centimeters, the angle the third side makes with the side measuring 8 centimeters, or the angle the third side makes with the side forming the 34° angle with the side measuring 8 centimeters

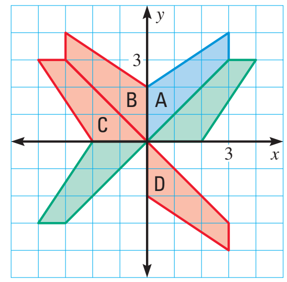
4. Yes; yes; $\angle ACD$ and $\angle BCE$ are vertical angles so they are congruent which makes $\triangle ACD \cong \triangle BCE$ by SAS. Since $\triangle ACD \cong \triangle BCE$, $\overline{AD} \cong \overline{BE}$ because corr. parts of $\cong$ △s are $\cong$.

5a. In $\triangle ABC$, it is given that $\overline{AB} \cong \overline{CB}$, therefore $\angle BCE \cong \angle BAE$ by the Base Angles Theorem.

5b. *Sample answer:* $\triangle ABE \cong \triangle CBE$ by AAS. $\overline{CE} \cong \overline{AE}$ because corr. parts of $\cong$ △s are $\cong$. $\triangle FAE \cong \triangle DCE$ by ASA and therefore $\overline{AF} \cong \overline{CD}$ because corr. parts of $\cong$ △s are $\cong$.

MIXED REVIEW *of Problem Solving*

1. MULTI-STEP PROBLEM Use the quilt pattern shown below.



a. Figure B is the image of Figure A. Name and *describe* the transformation. reflection in the *y*-axis

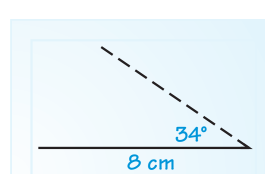
b. Figure C is the image of Figure A. Name and *describe* the transformation. rotation of 90° counterclockwise

c. Figure D is the image of Figure A. Name and *describe* the transformation. reflection in the *x*-axis

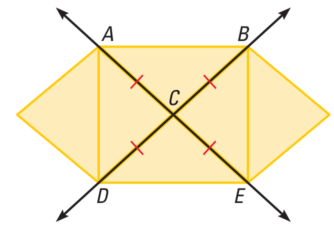
d. *Explain* how you could complete the quilt pattern using transformations of Figure A. Rotate Figure A 90° clockwise and 180°.

2. SHORT RESPONSE You are told that a triangle has sides that are 5 centimeters and 3 centimeters long. You are also told that the side that is 5 centimeters long forms an angle with the third side that measures 28°. Is there only one triangle that has these given dimensions? *Explain* why or why not. No; the given angle is not the included angle.

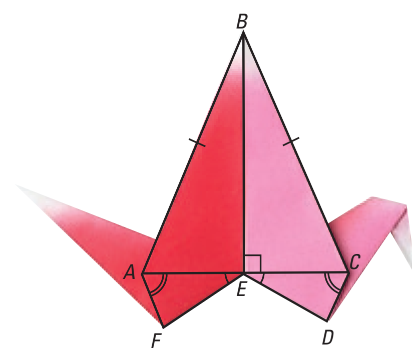
3. OPEN-ENDED A friend has drawn a triangle on a piece of paper and she is describing the triangle so that you can draw one that is congruent to hers. So far, she has told you that the length of one side is 8 centimeters and one of the angles formed with this side is 34°. *Describe* three pieces of additional information you could use to construct the triangle. See margin.



4. SHORT RESPONSE Can the triangles *ACD* and *BCE* be proven congruent using the information given in the diagram? Can you show that $\overline{AD} \cong \overline{BE}$? *Explain*. See margin.



5. EXTENDED RESPONSE Use the information given in the diagram to prove the statements below. a, b. See margin.



a. Prove that $\angle BCE \cong \angle BAE$.

b. Prove that $\overline{AF} \cong \overline{CD}$.

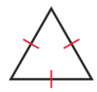
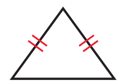
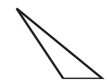
6. GRIDDED ANSWER Find the value of *x* in the diagram. 7

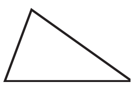
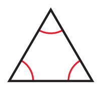
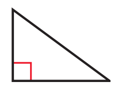
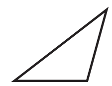
4 CHAPTER SUMMARY

BIG IDEAS — *For Your Notebook*

Big Idea 1

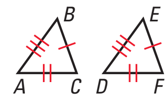
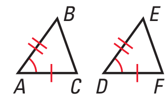
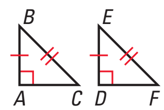
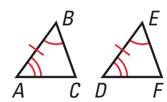
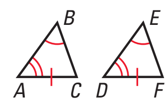
Classifying Triangles by Sides and Angles

	Equilateral	Isosceles	Scalene
Sides			
	3 congruent sides	2 or 3 congruent sides	No congruent sides

	Acute	Equiangular	Right	Obtuse
Angles				
	3 angles $< 90°$	3 angles $= 60°$	1 angle $= 90°$	1 angle $> 90°$

Big Idea 2

Proving That Triangles Are Congruent

SSS	All three sides are congruent.	$\triangle ABC \cong \triangle DEF$	
SAS	Two sides and the included angle are congruent.	$\triangle ABC \cong \triangle DEF$	
HL	The hypotenuse and one of the legs are congruent. (Right triangles only)	$\triangle ABC \cong \triangle DEF$	
ASA	Two angles and the included side are congruent.	$\triangle ABC \cong \triangle DEF$	
AAS	Two angles and a (non-included) side are congruent.	$\triangle ABC \cong \triangle DEF$	

Big Idea 3

Using Coordinate Geometry to Investigate Triangle Relationships

You can use the Distance and Midpoint Formulas to apply postulates and theorems to triangles in the coordinate plane.

Additional Resources

The following resources are available to help review the materials in this chapter.

Chapter Resource Book

- Chapter Review Games and Activities
- Cumulative Practice

Student Resources in Spanish

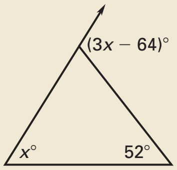
@HomeTutor

Vocabulary Practice

Vocabulary practice is available at **my.hrw.com**

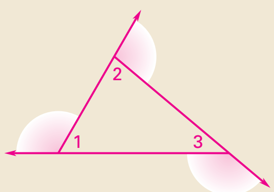
Extra Example 1

Find the measure of the exterior angle shown. **110°**



3. An isosceles triangle has at least two congruent sides while a scalene triangle has no congruent sides.

4.



4 CHAPTER REVIEW

@HomeTutor
my.hrw.com
- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see p. PT2.

- triangle
  scalene, isosceles, equilateral, acute, right, obtuse, equiangular
- interior angles
- exterior angles
- corollary to a theorem
- congruent figures
- corresponding parts
- rigid motions
- right triangle
  legs, hypotenuse
- flow proof
- isosceles triangle
  legs, vertex angle, base, base angles
- transformation
- image
- congruence transformation
  translation, reflection, rotation

VOCABULARY EXERCISES

1. Copy and complete: A triangle with three congruent angles is called _?_. **equiangular**
2. **WRITING** *Compare* vertex angles and base angles. **In an isosceles triangle, base angles are opposite the congruent sides while the congruent sides form the vertex angle.**
3. **WRITING** *Describe* the difference between isosceles and scalene triangles. **See margin.**

3. An isosceles triangle has at least two congruent sides while a scalene triangle has no congruent sides.

4. Sketch an acute scalene triangle. Label its interior angles 1, 2, and 3. Then draw and shade its exterior angles. **See margin.**
5. If $\triangle PQR \cong \triangle LMN$, which angles are corresponding angles? Which sides are corresponding sides?
   $\angle P$ and $\angle L$, $\angle Q$ and $\angle M$, $\angle R$ and $\angle N$; $\overline{PQ}$ and $\overline{LM}$, $\overline{QR}$ and $\overline{MN}$, $\overline{RP}$ and $\overline{NL}$

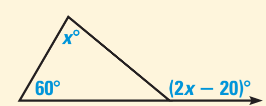
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of this Chapter.

4.1 Apply Triangle Sum Properties

EXAMPLE

Find the measure of the exterior angle shown.



Use the Exterior Angle Theorem to write and solve an equation to find the value of x.

$(2x - 20)° = 60° + x°$ **Apply the Exterior Angle Theorem.**

$x = 80$ **Solve for x.**

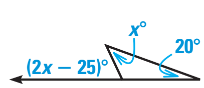
The measure of the exterior angle is $(2 \cdot 80 - 20)°$, or $140°$.

EXERCISES

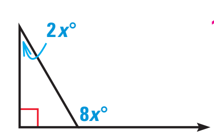
EXAMPLE 3 for Exs. 6–8

Find the measure of the exterior angle shown.

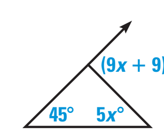
6. **65°**



7. **120°**



8. **90°**



@HomeTutor
my.hrw.com
Chapter Review Practice

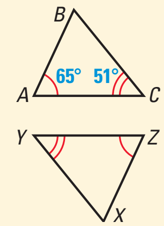
4.2 Apply Congruence and Triangles

EXAMPLE

Use the Third Angles Theorem to find $m\angle X$.

In the diagram, $\angle A \cong \angle Z$ and $\angle C \cong \angle Y$. By the Third Angles Theorem, $\angle B \cong \angle X$. Then by the Triangle Sum Theorem, $m\angle B = 180° - 65° - 51° = 64°$.

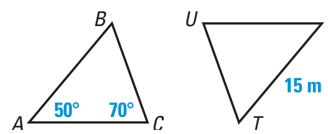
So, $m\angle X = m\angle B = 64°$ by the definition of congruent angles.



EXERCISES

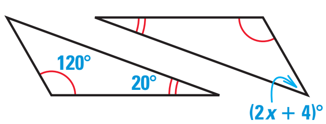
EXAMPLES 2 and 4 for Exs. 9–14

In the diagram, $\triangle ABC \cong \triangle VTU$. Find the indicated measure.

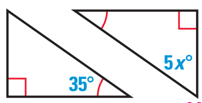


9. $m\angle B$ **60°**
10. AB **15 m**
11. $m\angle T$ **60°**
12. $m\angle V$ **50°**

Find the value of x.

13. 

18

14. 

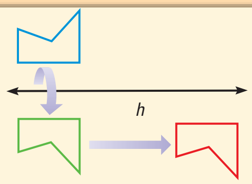
11

4.3 Relate Transformations and Congruence

EXAMPLE

Describe the transformation(s) you can use to move the blue figure onto the red figure.

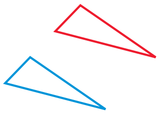
First reflect the figure over h. Then translate it right.

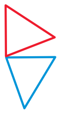


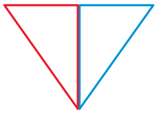
EXERCISES

EXAMPLES 1 and 2 for Exs. 15–18

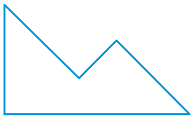
Describe the transformation(s) you can use to move the blue figure onto the red figure.

15. 

16. 

17. 

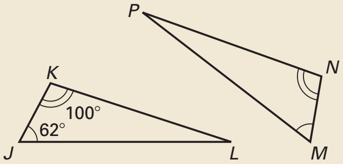
18. Copy the figure. Draw an example of the effect of a reflection on the figure.



15. translation up and right
16. rotation
17. reflection
18. Check students' drawings.

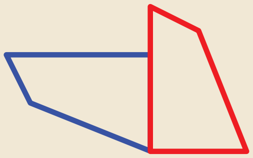
Extra Example 2

Use the Third Angle Theorem to find $m\angle P$. **18°**



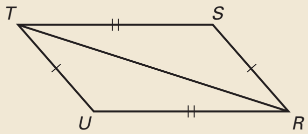
Extra Example 3

Describe the transformation(s) you can use to move the blue figure onto the red figure.



Extra Example 4

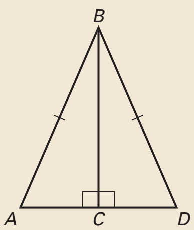
Prove that $\triangle RST \cong \triangle TUR$.



The marks show that $\overline{RS} \cong \overline{UT}$ and $\overline{ST} \cong \overline{UR}$. By the Refl. Prop. of Cong. Segs., $\overline{RT} \cong \overline{RT}$. So $\triangle RST \cong \triangle TUR$ by the SSS Cong. Post.

Extra Example 5

Prove that $\triangle ABC \cong \triangle DBC$.

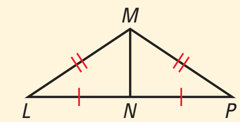


From the diagram, $\angle ACB$ and $\angle DCB$ are right angles and $\overline{AB} \cong \overline{DB}$. By the Refl. Prop. of $\cong$ Segs., $\overline{BC} \cong \overline{BC}$. Therefore, $\triangle ABC \cong \triangle DBC$ by the HL Cong. Thm.

4.4 Prove Triangles Congruent by SSS

EXAMPLE

Prove that $\triangle LMN \cong \triangle PMN$.



The marks on the diagram show that $\overline{LM} \cong \overline{PM}$ and $\overline{LN} \cong \overline{PN}$. By the Reflexive Property, $\overline{MN} \cong \overline{MN}$.

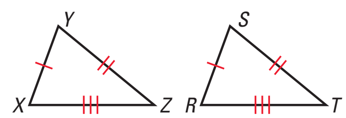
So, by the SSS Congruence Postulate, $\triangle LMN \cong \triangle PMN$.

EXERCISES

EXAMPLE 1 for Exs. 19–20

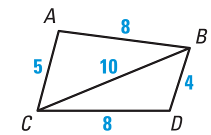
Decide whether the congruence statement is true. *Explain* your reasoning.

19. $\triangle XYZ \cong \triangle RST$



true; SSS

20. $\triangle ABC \cong \triangle DCB$

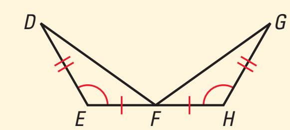


not true; $BD \neq CA$

4.5 Prove Triangles Congruent by SAS and HL

EXAMPLE

Prove that $\triangle DEF \cong \triangle GHF$.



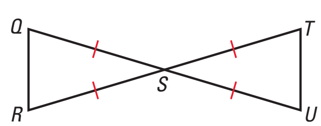
From the diagram, $\overline{DE} \cong \overline{GH}$, $\angle E \cong \angle H$, and $\overline{EF} \cong \overline{HF}$.
By the SAS Congruence Postulate, $\triangle DEF \cong \triangle GHF$.

EXERCISES

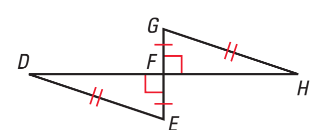
EXAMPLES 1 and 3 for Exs. 21–22

Decide whether the congruence statement is true. *Explain* your reasoning.

21. $\triangle QRS \cong \triangle TUS$ true; SAS



22. $\triangle DEF \cong \triangle GHF$ false; $\triangle DEF \cong \triangle HGF$ by HL

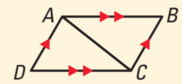


@HomeTutor
my.hrw.com
Chapter Review Practice

4.6 Prove Triangles Congruent by ASA and AAS

EXAMPLE

Prove that $\triangle DAC \cong \triangle BCA$.



By the Reflexive Property, $\overline{AC} \cong \overline{AC}$. Because $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$, $\angle DAC \cong \angle BCA$ and $\angle DCA \cong \angle BAC$ by the Alternate Interior Angles Theorem. So, by the ASA Congruence Postulate, $\triangle ADC \cong \triangle CBA$.

EXERCISES

EXAMPLES 1 and 2
for Exs. 23–24

State the third congruence that is needed to prove that $\triangle DEF \cong \triangle GHJ$ using the given postulate or theorem.

23. GIVEN ▶ $\overline{DE} \cong \overline{GH}$, $\angle D \cong \angle G$, _?_ $\cong$ _?_

Use the AAS Congruence Theorem. **$\angle F, \angle J$**

24. GIVEN ▶ $\overline{DF} \cong \overline{GJ}$, $\angle F \cong \angle J$, _?_ $\cong$ _?_

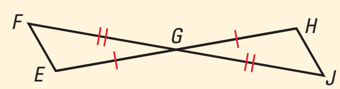
Use the ASA Congruence Postulate. **$\angle D, \angle G$**

4.7 Use Congruent Triangles

EXAMPLE

GIVEN ▶ $\overline{FG} \cong \overline{JG}$, $\overline{EG} \cong \overline{HG}$

PROVE ▶ $\overline{EF} \cong \overline{HJ}$



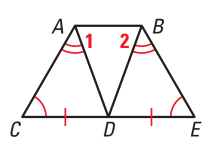
You are given that $\overline{FG} \cong \overline{JG}$ and $\overline{EG} \cong \overline{HG}$. By the Vertical Angles Congruence Theorem, $\angle FGE \cong \angle JGH$. So, $\triangle FGE \cong \triangle JGH$ by the SAS Congruence Postulate. Corresponding parts of $\cong$ ⚠ are $\cong$, so $\overline{EF} \cong \overline{HJ}$.

EXERCISES

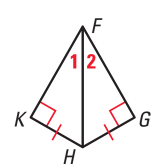
EXAMPLE 3
for Exs. 25–27

Write a plan for proving that $\angle 1 \cong \angle 2$. **21–23. See margin.**

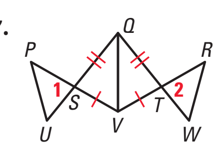
25.



26.

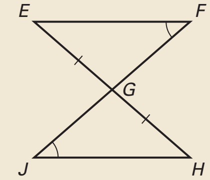


27.



Extra Example 6

Prove that $\triangle EFG \cong \triangle HJG$.

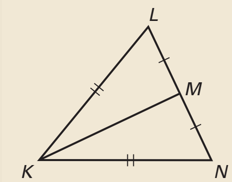


From the diagram, $\overline{EG} \cong \overline{HG}$ and $\angle F \cong \angle J$. $\angle EGF \cong \angle HGJ$ by the Vertical Angles Thm. Therefore, $\triangle EFG \cong \triangle HJG$ by the AAS Cong. Thm.

Extra Example 7

Given: $\overline{KL} \cong \overline{KN}$, $\overline{LM} \cong \overline{NM}$

Prove: $\angle LKM \cong \angle NKM$.



It is given that $\overline{KL} \cong \overline{KN}$ and $\overline{LM} \cong \overline{NM}$. By the Refl. Prop. of $\cong$ Segs., $\overline{KM} \cong \overline{KM}$. So, $\triangle KLM \cong \triangle KNM$ by the SSS Cong. Post. Corresponding parts of $\cong$ ⚠ are $\cong$, so $\angle LKM \cong \angle NKM$.

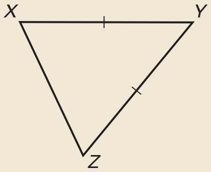
25. Show $\triangle ACD$ and $\triangle BED$ are congruent by AAS, which makes $\overline{AD}$ congruent to $\overline{BD}$. $\triangle ABD$ is then an isosceles triangle, which makes $\angle 1$ and $\angle 2$ congruent.

26. Show $\triangle FHK$ and $\triangle FHG$ are congruent using HL. $\angle 1$ and $\angle 2$ will be congruent because corr. parts of $\cong$ ⚠ are $\cong$.

27. Show $\triangle QVS$ congruent to $\triangle QVT$ by SSS, which gives $\angle QSV$ congruent to $\angle QTV$. Using vertical angles and the Transitive Property, you get $\angle 1$ congruent to $\angle 2$.

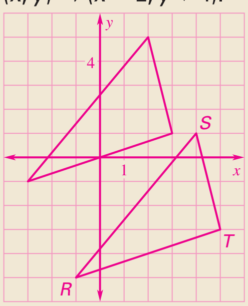
Extra Example 8

$\triangle XYZ$ is isosceles. Name two congruent angles. $\angle X \cong \angle Z$

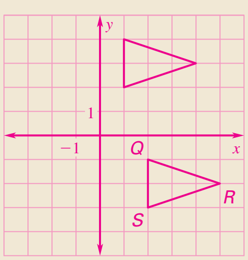


Extra Example 9

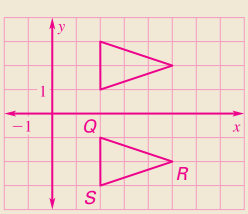
Triangle RST has vertices $R(-1, -5)$, $S(4, 1)$, and $T(5, -3)$. Sketch $\triangle RST$ and its image after the translation $(x, y) \rightarrow (x - 2, y + 4)$.



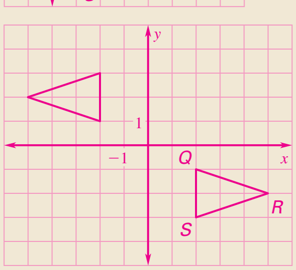
31.



32.



33.



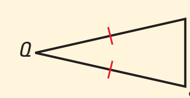
4 CHAPTER REVIEW

4.8 Use Isosceles and Equilateral Triangles

EXAMPLE

$\triangle QRS$ is isosceles. Name two congruent angles.

$\overline{QR} \cong \overline{QS}$, so by the Base Angles Theorem, $\angle R \cong \angle S$.

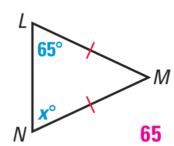


EXERCISES

EXAMPLE 3 for Exs. 28–30

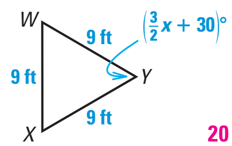
Find the value of x.

28.



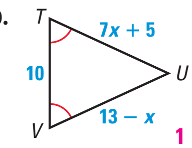
65

29.



20

30.



1

4.9 Perform Congruence Transformations

EXAMPLE

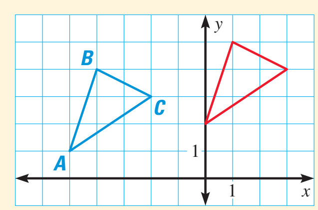
Triangle ABC has vertices $A(-5, 1)$, $B(-4, 4)$, and $C(-2, 3)$. Sketch $\triangle ABC$ and its image after the translation $(x, y) \rightarrow (x + 5, y + 1)$.

$(x, y) \rightarrow (x + 5, y + 1)$

$A(-5, 1) \rightarrow (0, 2)$

$B(-4, 4) \rightarrow (1, 5)$

$C(-2, 3) \rightarrow (3, 4)$



EXERCISES

EXAMPLES 2 and 3 for Exs. 31–33

Triangle QRS has vertices $Q(2, -1)$, $R(5, -2)$, and $S(2, -3)$. Sketch $\triangle QRS$ and its image after the transformation. 27–29. See margin.

31. $(x, y) \rightarrow (x - 1, y + 5)$

32. $(x, y) \rightarrow (x, -y)$

33. $(x, y) \rightarrow (-x, -y)$

Additional Resources

Assessment Book

- Chapter Test, Levels A, B, C
- Standardized Chapter Test
- SAT/ACT Chapter Test
- Alternative Assessment

ExamView™ Assessment Suite

Chapter Test

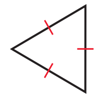
Easily-readable reduced copies of Chapter Test B, the Standardized Chapter Test, and the Alternative Assessment from the Assessment Book can be found at the beginning of this chapter.

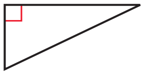
12. Statements (Reasons)

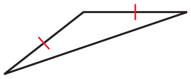
1. △*ABC* is isosceles with base $\overline{AC}$, $\overline{BD}$ bisects ∠*B*. (Given)
2. $\overline{AB} \cong \overline{BC}$ (Definition of isosceles triangle)
3. ∠*ABD* ≅ ∠*CBD* (Definition of angle bisector)
4. $\overline{BD} \cong \overline{BD}$ (Reflexive Property of Segment Congruence)
5. △*ABD* ≅ △*CBD* (SAS)

4 CHAPTER TEST

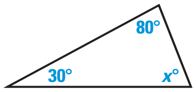
Classify the triangle by its sides and by its angles.

1.  equilateral, acute (or equiangular)

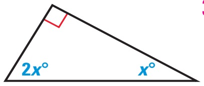
2.  scalene, right

3.  isosceles, obtuse

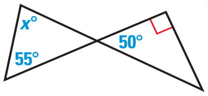
In Exercises 4–6, find the value of *x*.

4. 

70

5. 

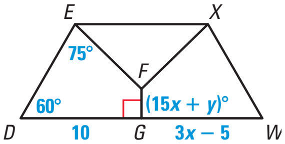
30

6. 

75

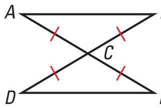
7. In the diagram, *DEFG* ≅ *WXFG*. Find the values of *x* and *y*. 5, 15

8. Use the art in Exercise 11. Describe the transformation(s) you can use to move *MNP* onto *PQM*.



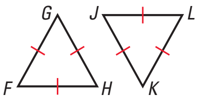
In Exercises 9–11, decide whether the triangles can be proven congruent by the given postulate.

9. △*ABC* ≅ △*EDC* by SAS



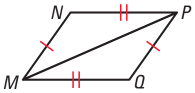
proven

10. △*FGH* ≅ △*JKL* by ASA



proven

11. △*MNP* ≅ △*PQM* by SSS

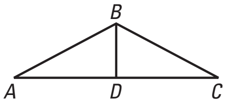


proven

12. Write a proof. See margin.

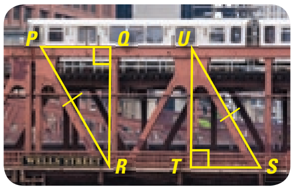
GIVEN ▶ △*ABC* is isosceles with base $\overline{AC}$, $\overline{BD}$ bisects ∠*B*.

PROVE ▶ △*ABD* ≅ △*CBD*

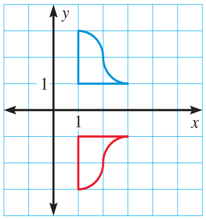


13. What is the third congruence needed to prove that △*PQR* ≅ △*STU* using the indicated theorem?

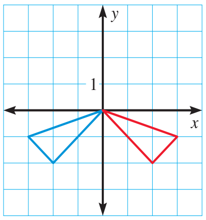
a. HL $\overline{QP} \cong \overline{TS}$ or $\overline{QR} \cong \overline{TU}$ **b.** AAS ∠*P* ≅ ∠*S* or ∠*R* ≅ ∠*U*



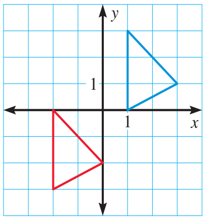
Decide whether the transformation is a *translation*, *reflection*, or *rotation*.

14. 

reflection

15. 

reflection

16. 

translation

4 ALGEBRA REVIEW

Animated Algebra
my.hrw.com

SOLVE INEQUALITIES AND ABSOLUTE VALUE EQUATIONS

EXAMPLE 1 Solve inequalities

Solve $-3x + 7 \le 28$. Then graph the solution.

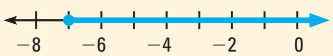
When you multiply or divide each side of an inequality by a *negative* number, you must reverse the inequality symbol to obtain an equivalent inequality.

$-3x + 7 \le 28$ — **Write original inequality.**

$-3x \le 21$ — **Subtract 7 from both sides.**

$x \ge -7$ — **Divide each side by -3. Reverse the inequality symbol.**

▶ The solutions are all real numbers greater than or equal to -7. The graph is shown at the right.



EXAMPLE 2 Solve absolute value equations

Solve $|2x + 1| = 5$.

The expression inside the absolute value bars can represent 5 or -5.

STEP 1 Assume $2x + 1$ represents 5.

$2x + 1 = 5$

$2x = 4$

$x = 2$

STEP 2 Assume $2x + 1$ represents -5.

$2x + 1 = -5$

$2x = -6$

$x = -3$

▶ The solutions are 2 and -3.

EXERCISES

EXAMPLE 1 for Exs. 1–12

Solve the inequality. Then graph the solution. **1–12. See margin for art.**

1. $x - 6 > -4$ $\mathbf{x > 2}$
2. $7 - c \le -1$ $\mathbf{c \ge 8}$
3. $-54 \ge 6x$ $\mathbf{x \le -9}$
4. $\frac{5}{2}t + 8 \le 33$ $\mathbf{t \le 10}$
5. $3(y + 2) < 3$ $\mathbf{y < -1}$
6. $\frac{1}{4}z < 2$ $\mathbf{z < 8}$
7. $5k + 1 \ge -11$ $\mathbf{k \ge -\frac{12}{5}}$
8. $13.6 > -0.8 - 7.2r$ $\mathbf{r > -2}$
9. $6x + 7 < 2x - 3$ $\mathbf{x < -\frac{5}{2}}$
10. $-v + 12 \le 9 - 2v$ $\mathbf{v \le -3}$
11. $4(n + 5) \ge 5 - n$ $\mathbf{n \ge -3}$
12. $5y + 3 \ge 2(y - 9)$ $\mathbf{y \ge -7}$

EXAMPLE 2 for Exs. 13–27

Solve the equation.

13. $|x - 5| = 3$ **2, 8**
14. $|x + 6| = 2$ **−8, −4**
15. $|4 - x| = 4$ **0, 8**
16. $|2 - x| = 0.5$ **1.5, 2.5**
17. $|3x - 1| = 8$ $\mathbf{-\frac{7}{3}, 3}$
18. $|4x + 5| = 7$ $\mathbf{-3, \frac{1}{2}}$
19. $|x - 1.3| = 2.1$ **−0.8, 3.4**
20. $|3x - 15| = 0$ **5**
21. $|6x - 2| = 4$ $\mathbf{-\frac{1}{3}, 1}$
22. $|8x + 1| = 17$ $\mathbf{-\frac{9}{4}, 2}$
23. $|9 - 2x| = 19$ **−5, 14**
24. $|0.5x - 4| = 2$ **4, 12**
25. $|5x - 2| = 8$ $\mathbf{-\frac{6}{5}, 2}$
26. $|7x + 4| = 11$ $\mathbf{-\frac{15}{7}, 1}$
27. $|3x - 11| = 4$ $\mathbf{\frac{7}{3}, 5}$

Extra Example 1

Solve $4x - 7 > 29$. Then graph the solution. $\mathbf{x > 9}$

Extra Example 2

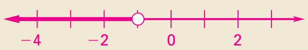
Solve $|5x - 6| = 9$. $\mathbf{3, -\frac{3}{5}}$

1.


2.


3.


4.

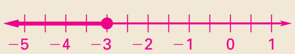

5.


6.

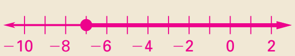

7.


8.


9.


10.


11.


12.


Test-Taking Strategy

Students should read and interpret any graph or diagram that is provided with the question. Often they will need additional information from the graph or diagram to answer the question. As they begin to solve the problem, have them refer to the graph or diagram to see what additional information they can find that will be helpful.

Avoiding Common Errors

In Problem 1 students may overlook the 5's on the axis and just count by 1's. Stress the importance of examining the graph to see what the scale is.

Teaching Strategy

For Problem 1, instead of computing all the distances, show students that for $\overline{JL}$ to be congruent to $\overline{AC}$ it must be the same number of gridlines on the graph. Count the gridlines between A and C and move to the right that same number from J. Then find the coordinates of this new point L.

Mathematical Reasoning

For Problem 2, suggest that students write another system of two equations they could use to solve the problem. Ask which system is easier to solve and why?

$2y + 3x - 4 = 180$, $4x - 47 + 2y = 180$; The system in the given solution is easier, because one of the equations has only one variable.

Graphing Calculator

For Exercise 1 students can put the ordered pairs they are given into L_1 and L_2 in the Stat menu of their graphing calculators. Then they can assign $L_3 = L_1 - 2$ and $L_4 = L_2 + 3$. The image coordinates will be in L_3 and L_4.

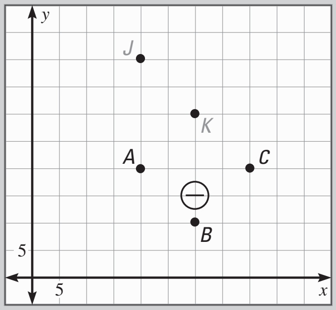
4 ★ Standardized TEST PREPARATION

CONTEXT-BASED MULTIPLE CHOICE QUESTIONS

Some of the information you need to solve a context-based multiple choice question may appear in a table, a diagram, or a graph.

PROBLEM 1

Five of six players on a lacrosse team are set up in a 2-3-1 formation. In this formation, the players form two congruent triangles. Three **attackmen** form one triangle. Three **midfielders** form the second triangle. In the diagram, where should player L stand so that $\triangle ABC \cong \triangle JKL$?



Ⓐ (8, 8) Ⓑ (20, 60)

Ⓒ (40, 40) Ⓓ (30, 15)

Plan

INTERPRET THE GRAPH Use the graph to determine the coordinates of each player. Use the Distance Formula to check the coordinates in the choices.

Solution

STEP 1 Find the coordinates of each vertex.

For $\triangle ABC$, the coordinates are $A(20, 20)$, $B(30, 10)$, and $C(40, 20)$. For $\triangle JKL$, the coordinates are $J(20, 40)$, $K(30, 30)$, and $L(\underline{\ ?\ }, \underline{\ ?\ })$.

STEP 2 Calculate BC and CA.

Because $\triangle ABC \cong \triangle JKL$, $BC = KL$ and $CA = LJ$. Find BC and CA.

By the Distance Formula, $BC = \sqrt{(40 - 30)^2 + (20 - 10)^2} = \sqrt{200} = 10\sqrt{2}$ yards.

Also, $CA = \sqrt{(20 - 40)^2 + (20 - 20)^2} = \sqrt{400} = 20$ yards.

STEP 3 Check the choices to find the coordinates that produce the congruent corresponding sides.

Check the coordinates given in the choices to see whether $LJ = CA = 20$ yards and $KL = BC = 10\sqrt{2}$ yards. As soon as one set of coordinates does not work for the first side length, you can move to the next set.

Choice A: $L(8, 8)$, so $LJ = \sqrt{(20 - 8)^2 + (40 - 8)^2} = 4\sqrt{73} \neq 20$ ✗

Choice B: $L(20, 60)$, so $LJ = \sqrt{(20 - 20)^2 + (40 - 60)^2} = \sqrt{400} = 20$ ✓

and $KL = \sqrt{(20 - 30)^2 + (60 - 30)^2} = \sqrt{1000} \neq 10\sqrt{2}$ ✗

Choice C: $L(40, 40)$, so $LJ = \sqrt{(20 - 40)^2 + (40 - 40)^2} = \sqrt{400} = 20$ ✓

and $KL = \sqrt{(40 - 30)^2 + (40 - 30)^2} = \sqrt{200} = 10\sqrt{2}$ ✓

Player L should stand at (40, 40). The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

Answers

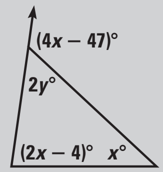
1. D
2. D

PROBLEM 2

Use the diagram to find the value of y.

(A) 15.5 (B) 27.5

(C) 43 (D) 82



Plan

INTERPRET THE DIAGRAM All of the angle measures in the diagram are labeled with algebraic expressions. Use what you know about the angles in a triangle to find the value of y.

Solution

STEP 1 Find the value of x.

Use the Exterior Angle Theorem to find the value of x.

$(4x - 47)° = (2x - 4)° + x°$	**Exterior Angle Theorem**
$4x - 47 = 3x - 4$	**Combine like terms.**
$x = 43$	**Solve for *x*.**

STEP 2 Find the value of y.

Use the Linear Pair Postulate to find the value of y.

$(4x - 47)° + 2y° = 180°$	**Linear Pair Postulate**
$[4(43) - 47] + 2y = 180$	**Substitute 43 for *x*.**
$125 + 2y = 180$	**Simplify.**
$y = 27.5$	**Solve for *y*.**

The correct answer is B. (A) (B) (C) (D)

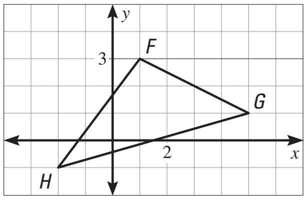
PRACTICE

1. In Problem 2, what are the measures of the interior angles of the triangle?

   (A) 27.5°, 43°, 109.5° (B) 27.5°, 51°, 86°

   (C) 40°, 60°, 80° (D) 43°, 55°, 82°

2. What are the coordinates of the vertices of the image of $\triangle FGH$ after the translation $(x, y) \rightarrow (x - 2, y + 3)$?



   (A) (3, 4), (−4, 4), (−1, 6)

   (B) (−2, −1), (1, 3), (5, 1)

   (C) (4, 1), (7, −1), (1, −3)

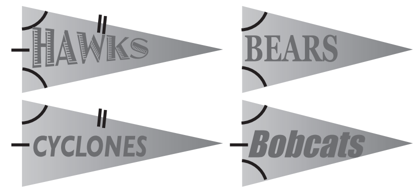
   (D) (−4, 2), (−1, 6), (3, 4)

Answers

1. B
2. A
3. C
4. C
5. C
6. C

MULTIPLE CHOICE

1. A teacher has the pennants shown below. Which pennants can you prove are congruent?



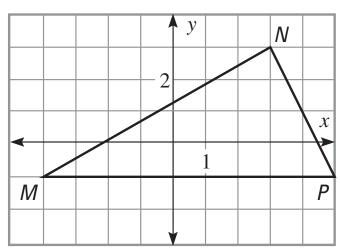
Ⓐ All of the pennants can be proven congruent.

Ⓑ The Hawks, Cyclones, and Bobcats pennants can be proven congruent.

Ⓒ The Bobcats and Bears pennants can be proven congruent.

Ⓓ None of the pennants can be proven congruent.

In Exercises 2 and 3, use the graph below.



2. What type of triangle is $\triangle MNP$?

Ⓐ Scalene

Ⓑ Isosceles

Ⓒ Right

Ⓓ Not enough information

3. Which are the coordinates of point Q such that $\triangle MNP \cong \triangle QPN$?

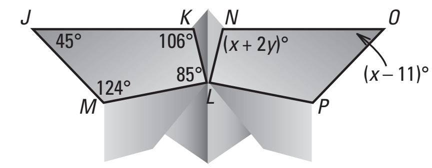
Ⓐ $(0, -3)$

Ⓑ $(-6, 3)$

Ⓒ $(12, 3)$

Ⓓ $(3, -5)$

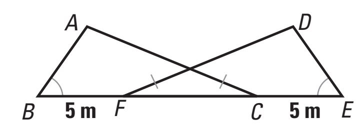
4. The diagram shows the final step in folding an origami butterfly. Use the congruent quadrilaterals, outlined in black, to find the value of $x + y$.



Ⓐ 25 Ⓑ 56

Ⓒ 81 Ⓓ 106

5. Which reason cannot be used to prove that $\angle A \cong \angle D$?



Ⓐ Base Angles Theorem

Ⓑ Segment Addition Postulate

Ⓒ SSS Congruence Postulate

Ⓓ Corresponding parts of congruent triangles are congruent.

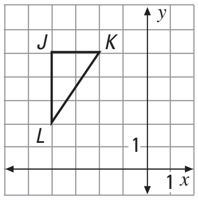
6. Which coordinates are the vertices of a triangle congruent to $\triangle JKL$?

Ⓐ $(-5, 0), (-5, 6), (-1, 6)$

Ⓑ $(-1, -5), (-1, -1), (1, -5)$

Ⓒ $(2, 1), (2, 3), (5, 1)$

Ⓓ $(4, 6), (6, 6), (6, 4)$

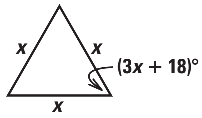


GRIDDED ANSWER

7. What is the perimeter of the triangle?

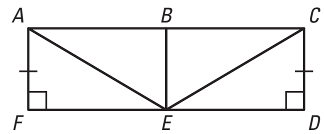
8. Figure *ABCD* has vertices $A(0, 2)$, $B(-2, -4)$, $C(2, 7)$, and $D(5, 0)$. What is the y-coordinate of the image of vertex B after the translation $(x, y) \rightarrow (x + 8, y - 0.5)$?

9. What is the value of x?



SHORT RESPONSE

10. If $\triangle ABE \cong \triangle EDC$, show that $\triangle EFA \cong \triangle CBE$.



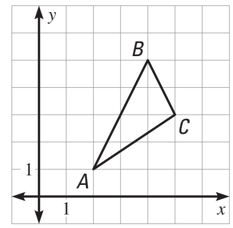
11. Two triangles have the same base and height. Are the triangles congruent? *Justify* your answer using an example.

12. If two people construct wooden frames for a triangular weaving loom using the instructions below, will the frames be congruent triangles? *Explain* your reasoning.

> Construct the frame so that the loom has a 90° angle at the bottom and 45° angles at the two upper corners. The piece of wood at the top should measure 72 inches.

EXTENDED RESPONSE

13. Use the diagram at the right.

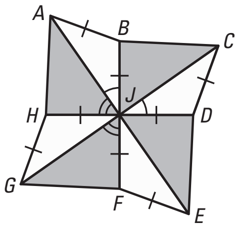


a. Copy the diagram onto a piece of graph paper. Reflect $\triangle ABC$ in the x-axis.

b. Copy and complete the table. *Describe* what you notice about the coordinates of the image compared to the coordinates of $\triangle ABC$.

	A	***B***	***C***
Coordinates of $\triangle ABC$	?	?	?
Coordinates of image	?	?	?

14. Kylie is designing a quilting pattern using two different fabrics. The diagram shows her progress so far.



a. Use the markings on the diagram to prove that all of the white triangles are congruent.

b. Prove that all of the shaded triangles are congruent.

c. Can you prove that the shaded triangles are right triangles? *Explain.*

7. 51

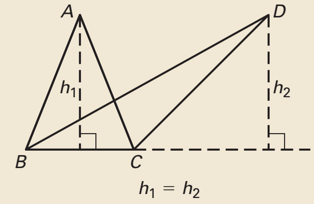
8. −4.5

9. 14

10. Statements (Reasons)

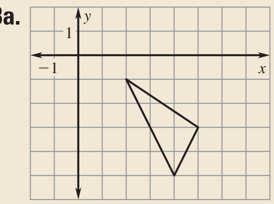
1. $\triangle ABE \cong \triangle EDC$, $\overline{FA} \cong \overline{DC}$, $\angle F$ and $\angle D$ are right angles. (Given)
2. $\angle ABE \cong \angle D$, $\overline{BE} \cong \overline{DC}$, $\overline{CE} \cong \overline{EA}$ (Corr. parts of $\cong$ △s are $\cong$.)
3. $m\angle ABE = m\angle D$ (Definition of congruent angles)
4. $m\angle ABE = 90°$ (Definition of right angle)
5. $m\angle CBE = 90°$ (Definition of linear pair)
6. $\angle CBE$ is a right angle (Definition of right angle)
7. $\triangle CBE$ is a right triangle. (Definition of right triangle)
8. $\overline{FA} \cong \overline{BE}$ (Transitive Property of Congruence)
9. $\triangle EFA \cong \triangle CBE$ (HL)

11. No; *sample diagram:*



12. yes; ASA

13a.



13b. $A(2, 1)$, $B(4, 5)$, $C(5, 3)$; $A'(2, -1)$, $B'(4, -5)$, $C'(5, -3)$; the x-coordinates are the same but the y-coordinates are opposites.

14a. The triangles are isosceles, so the base angles are congruent, and the triangles are congruent by AAS.

14b. Because corr. parts of $\cong$ △s are $\cong$, the triangles are congruent by SAS.

14c. No; there is not enough information to establish a right angle.